ESTIMATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT TEST DATA

By

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DEPRTMENT OF AERONAUTICAL ENGINEERING

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DEDICATED

TO THOSE, PASSED UNKNOWN
SERVING THE CAUSE OF
AVIATION

B.S.Y.

CERTIFICATE

This is to certify that the work " ESTIMATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT TEST DATA" has been carried out under my superivision and has not been submitted elsewhere for a degree.

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SUMMARY

In the present work, the stability and control derivatives have been determined from the flight test data. For extraction of aircraft parameters measured values of test inputs and the resulting output responses are required. The measured responses are compared with the calculated responses by assuming initial values of the parameters. The square of the difference between the measured and calculated responses is minimized by the modified Newton Raphson method and the Davidon-Fletcher-Powell method. The values of the parameters are updated at each iteration and the process is continued till the minima is achieved. The results obtained for a light subsonic aircraft by the two methods have been compared. It is found that all the important derivatives extracted by the two methods, are in close agreement. The computed time histories by the two methods match well with the flight data.

LIST OF SYMBOLS

A	stability matrix (PXP)
$A_{a_{ij}}$	null matrix except for the i-j^{th} element which equals (PXP)
A_i	i^{th} row of the stability matrix (1XP)
a_{ij}	i-j^{th} element of A
B	control matrix (PXQ)
$B_{b_{ij}}$	null matrix except for the i-j^{th} element which equals (PXQ)
B_i	i^{th} row of the control matrix (1XQ)
b_{ij}	i-j^{th} element of B
b	span m (ft)
C	augmented A and B matrices (PX(P+Q))
C_i	i^{th} row of the C matrix (1X(P+Q))
c	vector of unknown coefficients (mX1)
c_i	i^{th} element of the c vector
c_{m_α}	variation of pitching moment coefficient with angle of attack, rad^{-1}
$c_{m_{\delta_e}}$	variation of pitching moment coefficient with elevator angle, deg^{-1} or rad^{-1}
c_T	vector of actual values of unknown coefficients ([illegible]X1)
D_1	weighting matrix for observation vector (RXR)
G	partition of matrix relating the state vector to the observation vector (R-P)XP)
g	acceleration due to gravity, $m/sec^2 (ft/sec^2)$

g	vector of observation biases (RX1)
g_i	i^{th} element of g
H	partition of matrix relating the control vector to the observation vector (R-P)XQ)
I	identity matrix
I_{xx}, I_{yy}, I_{zz}	moment of inertia about x, y and z axes
J	cost functional or weighted mean-square-fit error
l	number of time samples
m	number of unknowns in c vector
P	number of state variables
p	roll rate, rad/sec or deg/sec
Q	number of control variables
q	pitch rate, rad/sec or deg/sec
R	number of observation variables
r	yaw rate, rad/sec or deg/sec
T	total time, sec
t	intermediate or incremental time, sec
u	control vector (QX1)
V	velocity, m/sec (ft/sec)
x	state vector (PX1)
x_i, x_j	i^{th} and j^{th} component of x
Y	side force divided by mass and velocity, rad/sec
y	observation vector (RX1)
y_i	i^{th} element of the y vector

Z	measurement of state variables (PX1)
$\dot{Z}$	measurement related to derivatives of state variables ((R-P)X1)
z	measurement of observation vector (RX1)
α	angle of attack of X-axis, rad or deg
β	angle of sideslip, rad or deg
Δ	increment
δ	first variation
δ_a	aileron deflection, rad or deg
δ_e	elevator deflection, rad or deg
δ_r	rudder deflection, rad or deg
τ	auxiliary time variable, sec
ϕ	bank angle, rad or deg
ω	frequency, rad/sec
$\nabla_c(\cdot)$	gradient of $(\cdot)$ with respect to c
$\nabla_g(\cdot)$	gradient of $(\cdot)$ with respect to g

Dimensional stability and control derivatives:

L_p	Dimensional variation of rolling moment with roll rate, sec^{-1}
L_r	Dimensional variation of rolling moment with yaw rate, sec^{-1}
L_β	Dimensional variation of rolling moment with side slip angle, sec^{-2}
N_p	Dimensional variation of yawing moment with roll rate, sec^{-1}
N_r	Dimensional variation of yawing moment with yaw rate, sec^{-1}

N_β Dimensional variation of yawing moment with side slip angle, sec^{-2}

Y_p Dimensional variation of side force with roll rate, $m/sec^2 (ft/sec^2)$

Y_β Dimensional variation of side force with side slip angle, $m/sec^2 (ft/sec^2)$

L_{δ_a} Dimensional variation of rolling moment with aileron angle, sec^{-2}

L_{δ_r} Dimensional variation of rolling moment with rudder-angle, sec^{-2}

N_{δ_a} Dimensional variation of yawing moment with aileron angle, sec^{-2}

N_{δ_r} Dimensional variation of yawing moment with rudder angle, sec^{-2}

Subscripts:

i i^{th} row or component

j j^{th} column or component

k iteration index

me measured

Superscripts:

i,j index representing time of sample

T matrix transpose

A dot over a quantity denotes the time derivative of that quantity. Principal axes are used throughout.

LIST OF FIGURES

FIG. NO.

CHAPTER 1

INTRODUCTION

1.1 GENERAL:

Over the past several years, there had been a renewed interest in determining dynamic aircraft parameters such as stability and control derivatives, from flight test measurements. The need for these data has long persisted. However, only recently highly automated data acquisition systems and advanced estimating techniques have been available, that one can extract such information efficiently.

Various methods of estimating aircraft parameters, i.e. stability and control derivatives, by use of wind tunnel measurements or from theory, are in existence. These derivatives can be used to compute time histories of various motions of the aircraft. It is important to realize that the computed motions and stability are meaningless, unless the equations of motion and stability derivatives are truly representative of the aircraft under consideration. Wind tunnel measurements are usually made with small models, and Reynolds number, roughness, tunnel wall effects, Mach number etc., generally are not properly scaled to simulate the full aircraft. It, therefore, becomes necessary to assess the correctness of the derivatives, so determined by the wind tunnel measurements. One

hod is to obtain the derivatives by flight testing of 'craft. In this method from a given group of measurements velocities, positions and accelerations taken over a time ;erval, we have to determine whether the stability and itrol derivatives can be estimated. The process of extracting aerical values for the aerodynamic stability and control :ivatives, is known as aircraft parameter identification.

The following are the needs for identification:-

To provide input information to aircraft simulators.

To provide basis for design of flight control systems.

The stability and control derivatives define a given rcraft more uniquely than the response mode criteria (As sted in various flying qualities specifications like MIL-F-85). It is thus anticipated that in future these parameters ll ultimately play a major role in the design, testing and ertification of aircraft.

3 BACK-GROUND AND BRIEF HISTORICAL DEVELOPMENT:

One of the first flight test programmes to obtain antitative measurements of aircraft aerodynamic characteris-cs was reported by Norton in 1919. Lift and drag coefficients re determined by equating the lift to weight and drag to e thrust.

Soule and Wheatley (e.g.1) appear to be the first to have determined all the major longitudinal stability and control derivatives of an aircraft from flight test data. This analysis used simplified equations representing one-degree-of freedom of motion. Equations were solved for one parameter at a time, assuming values for other parameters based on wind-tunnel tests. This basic approach was used till middle of 1940's.

Milliken (1947) pointed out that the increasing use of automatic control systems required more accurate modelling of the aircraft dynamic characteristics. These factors coupled with the research engineer's motivation to improve the accuracy of flight results, stimulated the development of several new techniques for determining stability and control derivatives from flight data.

In the late 1940's through mid 1950's servomechanism theory was expanded rapidly and the frequency-domain techniques of Nyquist and Bode (see e.g.2,3) were popular. A disadvantage of this approach was the considerable flight time required to sweep through all the frequencies of interest at each flight condition.

The problem of determining stability and control derivatives is based on a linearized, small-perturbation model of the aircraft dynamics. Hence it was natural to consider using

a linear least-square fit of flight data to the linearized equations of motion as was done by Greenberg(4) in 1951. In 1954 Shinbrot (5) developed a generalized least-square method which encompassed the earlier least-square methods. During the period when Greenberg and Shinbrot developed their methods, the digital computers were not available. These two methods required extensive calculations to match the computed response with the flight measured response of the aircraft. Thus a large volume of flight data had to be processed manually for extracting the stability and control derivatives.

'Analog Matching' (see e.g. 6,7) technique was used as early as 1951 to check aircraft parameters determined by other methods. Even though the technique of analog matching has greatly improved over the period, the accuracy of the results is highly dependent on the skill of an individual operator. If several parameters are to be determined, this technique requires excessive number of man-hours to obtain an acceptable solution.

Although several attempts were made throughout the 1950's and early 1960's to improve techniques, the effort was relatively small. Two factors caused a revolution in aircraft parameter estimation techniques starting in mid-1960's. These were:

1. Highly automated data acquisition systems were becoming standard in flight testing.

2. Large-capacity, high-speed digital computers were available to solve complicated algorithms efficiently. The ability to transfer the flight data directly to the computer with no manual operations on the data, helped in increasing the use of flight-testing techniques for evaluating aircraft parameters.

The interest in parameter estimation was renewed in 1968. Larson (8) applied the method of quasi-linearization at The Cornell Aeronautical Laboratory. Taylor and Iliff (9) used basically the same approach, but referred to it as the modified Newton-Raphson technique, at the NASA Flight Research Centre. Denery (10) applied the general theory of system identification to air vehicles for extracting aircraft parameters. Mehra (11) used the method of stochastic approximations to solve system identification problems. Later on the same approach was applied for aircraft identification (see 13).

1.4 AIRCRAFT PARAMETER ESTIMATION:

The general problem of parameter estimation is to determine certain characteristics of the physical system from experimental test data. Measurements are made of the test inputs and the resulting output responses that depend in some way on the system characteristics to be determined. Parameter estimation technique is the process of estimating characteristic from the input/output measurements.

The general concept of the aircraft parameter estimation technique is illustrated in Fig. 1. The various aspects of the problem are:

(i) Mathematical Model

(ii) Estimation Criterion

(iii) Computational Algorithm

(iv) Instrumentation and Data Acquisition

(v) Test Inputs

1.4.1 Mathematical Model:

A model must be selected that adequately represents the aircraft characteristics to be measured. For the present problem the aircraft is assumed to be rigid and the linearized equations of motion are considered. However, in other instances a more complex model may be necessary, such as at very high angles of attack a non-linear model may be required. An inappropriate model can degrade the accuracy of the parameter estimates.

1.4.2 Estimation Criterion:

There must be some means of assessing fit of the computed response to the measured response. To implement this 'criterion function' is used. It is usually some form of the integral square of the error between computed and measured response. The best estimate of the parameters is the set of parameters that minimizes the criterion function.

1.4.3 Computational Algorithm:

The criterion function is often non-linear with respect to the parameters to be estimated; therefore it has to be minimized by an iterative computational algorithm. Important factors in selecting the minimization algorithm are convergence, computation efficiency, local minima etc.

1.4.4. Instrumentation and Data Acquisition System:

Aircraft parameter estimation is highly dependent on the quality of the flight measured data. In reality bias and random errors can arise from improper location or orientation of sensors, calibration of measurement and recording system. Other errors can be introduced from electrical noise, engine vibrations, inappropriate signal filters, air turbulence etc. Any elimination of errors, noise or uncertainties within the data acquisition system will improve the accuracy of the estimates. A comprehensive discussion of flight-test instrumentation for aircraft parameter estimation is available in Ref. 12.

1.4.5 Test Input:

As a minimum requirement, the test input must excite the principal response modes that depend on the parameters to be determined. In the example considered in this work, a combination of rudder and aileron pulses adequately excited the

lateral-direction motion. However, a question arises whether one type of control input might be better than the other, in the sense that it provides better estimates. Several papers have considered this question (13) but the concept has not been fully explored in a flight-test application (14).

1.4 PRESENT WORK:

In the present work, we are interested in analysing the flight data, in order to obtain the stability and control derivatives. The details of flight data acquisition and those of test inputs have not been considered here. A subsonic light aircraft is selected for the analysis in the lateral-directional mode, as the flight data for the same could be obtained from Ref. 17.

The process of extraction of stability and control derivatives involves the measurement of test inputs and the resulting output responses. The measured responses are compared with the calculated responsesby assuming initial values of the parameters. The square of the difference between the measured and calculated responses is minimized by the modified Newton Raphson method and the Davidon-Fletcher-Powell method. The values of the parameters are updated at each iteration and the process is continued till the minima is achieved.

There is no direct reference in the literature about the use of the Davidon-Fletcher-Powell method for the flight dynamics use and its results. It was, therefore, decided to use this method to find out if it could be applied for solving aircraft dynamics problems.

The details of the methods for determination of stability and control derivatives from flight data are presented in Chapter 2. The identification problem is stated and a brief review of the various methods used for aircraft parameter identification has been carried out. The modified Newton Raphson method and the Davidon-Fletcher-Powell minimization techniques are discussed in detail.

The computational details, results, discussions and conclusions are given in Chapter 3. The parameters estimated by the two methods and the time histories are compared. Finally the source of errors are pointed out and the suggestions are made for further work.

CHAPTER 2

METHODS FOR DETERMINATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT DATA

2.1 INTRODUCTION:

A general statement of the problem of aircraft parameter identification, as applied to the extraction of lateral-directional stability and control derivatives, is presented here. A brief review of the previous methods is presented first. Subsequently, the two techniques chosen for the present study viz. the modified Newton Raphson technique and the Davidon-Fletcher-Powell minimization techniques are described in details.

2.2 STABILITY DERIVATIVE DETERMINATION: STATEMENT OF THE PROBLEM

For most of the aircraft dynamics problems, the longitudinal mode is usually seperated from the lateral-direction mode, because the resulting error is small. The estimation of stability and control derivatives in lateral-direction mode is more complex than the longitudinal mode. This work deals with the problem of stability and control derivative estimation in the lateral-direction mode.

The mathematical model often used to describe lateral-

directional airplane dynamics can be expressed as a system of linear ordinary differential equations with constant-coefficients, in the following form (see Appendix A)

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.1}$$

where

$$\dot{x}(t) = \begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} \quad x(t) = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} \quad u(t) = \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix}$$

The last column of matrix B represents the effect of uncertain bias on the measurement of $\dot{p}$, $\dot{r}$ and $\dot{\beta}$.

Let $z(t)$ and $\dot{z}(t)$ be the noise contaminated measurements of $x(t)$ and $\dot{x}(t)$ respectively. The control inputs $u(t)$ are considered to be noise-free.

Given z, $\dot{z}$ and u, the problem is to determine certain unknown elements of matrices A and B, i.e. the stability and control derivatives.

2.3 PREVIOUS METHODS : BRIEF REVIEW:

Some of the methods used earlier for determining stability and control derivatives are described below:

2.3.1. Simplified Equations Method:

For selected type of responses, the effect of only a few coefficients dominates, thus permitting the use of simple expressions to determine these coefficients. Wolowicz (12) obtained good approximations for some of the longitudinal stability derivatives by keeping only the dominant terms when equations of motion have been solved for a particular derivative. For example, $C_{m_{\delta_e}}$ can be determined from the initial portion of rapid pulse maneuver by :

$$C_{m_{\delta_e}} \simeq \frac{I_{yy}}{\frac{1}{2}\rho V^2 \, SC} \; \frac{\Delta \dot{q}}{\Delta \delta_e} \tag{2.2}$$

Similarly, C_{m_α} can be approximated to within 3% accuracy from the relation,

$$C_{m_\alpha} \simeq -\frac{I_{yy}}{\frac{1}{2}\rho V^2 \, SC} \; W^2_{n_{s.p.}} \tag{2.3}$$

where $W_{n_{s.p}}$ is the frequency of short period oscillations

Wolowicz (12) also gave some simplified approximations for the lateral stability derivatives. However, because the more complex behaviour of the airplane and large number of derivatives involved, the lateral-directional control and stability derivatives are not as readily and reliably determined by the use of approximate equations, as are the longitudinal derivatives.

The disadvantages of this method are:

1. Only some of the primary unknown coefficients of stability and control derivatives can be determined.

2. The forms of response that can be analyzed are very restrictive i.e. effect of control must either be dominant or negligible.

2.3.2. <u>Analog-Matching Method:</u>

In analog matching technique, the values of stability and control derivatives are assumed and motions are computed for the same control inputs as those used in flight tests. Comparisons are made between the computed and flight-measured responses. If any of the responses do not compare favourably, some of the derivatives are changed and the motions are computed again. The process is repeated until satisfactory

agreement is obtained between the estimated and flight-measured responses. The ability to converge to a set of acceptable derivatives, or of even finding acceptable agreement between computed and flight motions, is largely a matter of experience.

The main disadvantages of this technique are:

1. Analog-matching depends quite heavily on the experience of the operator. However, this method can be used as a back-up method and has given satisfactory results in many cases.
2. The method works successfully only when a single control surface is moved at a time so that the maneuvers are simple. (see 12).
3. When the maneuvers are made with stability augmentation system, the data is difficult to analyze.
4. This method is extremely time consuming.

2.3.3 Least-Squares Method:

Least-squares method assumes a performance criterion which minimizes the square of the state equation error, by substituting the measured value of the state and its derivatives i.e. it minimises the following cost functional:

$$J = \int_0^T (\dot{z} - Az - Bu)^T (\dot{z} - Az - Bu)\, dt \qquad (2.4)$$

To derive expressions for the values of A and B that minimizes Eq. (2.4), let

$$C = [\, A \,\vdots\, B \,]$$

Then,

$$J = \int_0^T \dot{z}^T \dot{z} \, dt - 2 \int_0^T \dot{z}^T c \left[- \frac{z}{u} - \right] dt + \int_0^T \left[- \frac{z}{u} -\right] c^T c \left[- \frac{z}{u} -\right] dt \qquad (2.5)$$

The minimization is achieved by taking the first derivative of Eq. (2.5) w.r.t. c i.e.

$$\frac{\partial J}{\partial c} = 0$$

This yields,

$$c^T = \left\{ \int_0^T \left[- \frac{z}{u} -\right] \left[- \frac{z}{u} -\right]^T dt \right\}^{-1} \int_0^T \left[- \frac{z}{u} -\right] \dot{z}^T \, dt \qquad (2.6)$$

Eq. (2.6) is the desired solution by the method of least-squares. It has the advantage of compact form but disguises the independence of each of the equations to be minimized. This can be shown in the following manner. Consider only the first state equation from Eq. (2.1),

$$\dot{x}_1(t) = A_1 \, x(t) + B_1 \, u(t)$$

where

$$A_1 = [\,L_p \;\; L_r \;\; L_\beta \;\; 0\,]$$

$$B_1 = [\,L_{\delta_a} \;\; L_{\delta_r} \;\; L_o\,]$$

$$C_1 = [\,A_1 \,\vdots\, B_1\,]$$

Then

$$J = \int_0^T [\,\dot{p}_{m_e} - L_p\, p_{m_e} - L_r\, r_{m_e} - L_\beta\, \beta_{m_e} - L_{\delta_a}\cdot\delta_a$$

$$- L_{\delta_r}\cdot\delta_r - L_o\,]^2\, dt$$

$$= \int_0^T \left\{ \dot{p}_{m_e} - c_1 \left[- \frac{z}{u} -\right] \right\}^2 dt$$

where the subscript ϕ_{m_e} denotes the measured value. Once again taking the first derivative and setting the resulting expression to zero, gives

$$c_1^T = \left\{ \int_0^T \left[- \frac{z}{u} -\right] \left[- \frac{z}{u} -\right]^T dt \right\}^{-1} \int_0^T \dot{p}_{m_e} \left[- \frac{z}{u} -\right] dt \tag{2.7}$$

Now c_1 is the first row of the matrix c and $\dot{p}_{m_e}$ is the first element of $\dot{z}$, which makes it apparent that the elements of the first row of the c matrix are independent of all the elements of $\dot{z}$ except the first, $\dot{p}_{m_e}$. A similar relationship

can be easily shown for the other rows of c matrix.

The disadvantages of this method are:

1. The row independence (shown above by Eq. 2.7) is one of the drawbacks of this method, in that only one of the measured state variables is used in determining a given row of c matrix. If one of the signals has not been measured, the least-squares method does not provide an estimate of the derivatives related to that signal.

This independence also illustrates that the estimate of one row of the c matrix is obtained independently of the other rows, and no 'trade-off' can be made between elements in different rows to improve the match.

2. If two or more of the measured responses are linearly related, this method gives an absurd solution or no solution (15).

3. This method gives excessive variance of the estimated coefficients (16).

2.4 MODIFIED NEWTON-RAPHSON AND DAVIDON-FLETCHER-POWELL TECHNIQUES:

2.4.1 General:

These techniques are means of selecting those parameter values which best fit an assumed model to a data set

according to a particular error criterion. The error criterion is more general than the least-squares criterion in that it permits the fit error to p, r, β and ϕ to be minimised as well as the fit error to $\dot{p}$, $\dot{r}$ and $\dot{\beta}$. These techniques also enable one to use a priori values of the stability derivatives, bias terms, and initial conditions to improve the fit of the equations to match flight test data.

The mathematical model chosen here describes the lateral-directional motions of an aircraft by linear, constant coefficient differential equations. Consider the following model (see Appendix A)

$$\dot{x} = Ax + Bu \tag{2.8}$$

and

$$y = \left[- \frac{I}{G} - \right] x + \left[- \frac{O}{H} - \right] u \tag{2.9}$$

where,

The vector y is the set of out-put response quantities:

$$y = \left[p \ r \ \beta \ \phi \ \dot{p} \ \dot{r} \ \dot{\beta} \right]^T$$

The matrices A and B are defined in Eq. (2.1)

$[I]$ = Identity Matrix

$[0]$ = Null matrix

$$G \doteq \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \end{bmatrix}$$

$$H = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \end{bmatrix}$$

Let z denote the measurement of the actual aircraft response quantities corresponding to the computed output response quantities, y. The measured response z would not be exactly the same as y because of,

(i) The measurement errors

(ii) The difference between the actual and the assumed linear mathematical model, Eq. (2.9).

The objective is to minimize the difference between z and The appropriate cost functional is,

$$J = \int_0^T (z-y)^T D_1 (z-y) \, dt \tag{2.10}$$

Where D_1 is a weighting matrix reflecting the relative confidence in the measurements.

After specifying the cost functional J, an algorithm is to be chosen for minimizing this cost functional. Many methods are available for non-linear minimization. However, for this particular work a modified Newton-Raphson technique due to Iliff and Taylor (9) and the Davidon Fletcher Powell technique are selected. They are described below.

2.4.2 Modified Newton-Raphson Minimization Technique

For convenience, let us define a column vector c of the unknowns to be estimated. The elements of c are some or all of the unknown elements of A and B of the noise biases g and of the initial conditions $x_i(o)$, e.g.

$$c = c\left[a_{ij},\ b_{ij},\ g_i,\ x_i(o)\right]$$

In all subsequent calculations, the noise biases g_i and the variable initial condition $x_i(o)$ have been neglected.

The Newton-Raphson technique is an iterative method for finding a zero of a nonlinear function of several parameters, or in this instance, a zero of the gradient of the cost functional J, i.e.

$$\nabla_c J = 0$$

Consider a two-term Taylor's series expansion of $\nabla_c J$ about the k^{th} value of c_k:

$$(\nabla_c J)_{k+1} \simeq (\nabla_c J)_k + (\nabla_c^2 J)_k \, \Delta c_{k+1} \tag{2.11}$$

where:

$$\Delta c_{k+1} = (c_{k+1} - c_k)$$

and $(\nabla_c^2 J)_k$ is the second gradient of the cost functional w.r.t. c, at the k^{th} iteration. If equation (2.11) is a sufficiently close approximation, the change in c on the $(k+1)^{th}$ iteration to make $(\nabla_c J)_{k+1}$ approximately zero is,

$$\Delta c_{k+1} = - \left[(\nabla_c^2 J)_k \right]^{-1} (\nabla_c J)_k \tag{2.12}$$

This is Newton-Raphson algorithm.

To evaluate the change in parameter value Δc_{k+1} by Eq. (2.12), the values of $(\nabla_c J)_k$ and $(\nabla_c^2 J)_k$ are needed. The gradient of J with respect to the vector c can be expressed in terms of the gradient of (z-y) by differentiating Eq. (2.10) as,

$$\nabla_c J = 2 \left\{ \int_0^T (z-y)^T D_1 \nabla_c (z-y) \, dt \right\}^T \tag{2.13}$$

In order to evaluate $\nabla_c J$ the term $\nabla_c (z-y)$ is required to be specified in addition to the terms already defined. This can be obtained by differentiating Eq.(2.9),

as follows:

$$\nabla_c (z-y) = -\left\{\nabla_c \left[-\frac{I}{G}-\right]\right\} x - \left[-\frac{I}{G}-\right]\nabla_c x - \left\{\nabla_c \left[-\frac{O}{H}-\right]\right\} u$$

$$- \left[-\frac{O}{H}-\right]\nabla_c u \qquad (2.14)$$

The quantity $\nabla_c (z-y)$ can be expressed in terms of various partial derivatives. These partial derivatives with respect to the individual coefficients of c (a_{ij}, b_{ij}) are,

$$\frac{\partial \left|-\frac{I}{G}-\right|}{\partial a_{ij}} x = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ x_j \\ 0 \\ 0 \end{bmatrix} \qquad \frac{\partial \left|-\frac{O}{H}-\right|}{\partial a_{ij}} u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{\partial \left|-\frac{I}{G}-\right|}{\partial b_{ij}} x = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \frac{\partial \left|-\frac{O}{H}-\right|}{\partial b_{ij}} u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ u_j \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

where x_j and u_j appear in i+p row of the vector.

The gradient of x with reference to c is given by

$$\nabla_c x = \begin{bmatrix} \frac{\partial p}{\partial c_1} & \frac{\partial p}{\partial c_2} & \cdots & \frac{\partial p}{\partial c_m} \\ \frac{\partial r}{\partial c_1} & \frac{\partial r}{\partial c_2} & \cdots & \frac{\partial r}{\partial c_m} \\ \frac{\partial \beta}{\partial c_1} & \frac{\partial \beta}{\partial c_2} & \cdots & \frac{\partial \beta}{\partial c_m} \\ \frac{\partial \phi}{\partial c_1} & \frac{\partial \phi}{\partial c_2} & \cdots & \frac{\partial \phi}{\partial c_m} \end{bmatrix}$$

The elements of gradient x can be determined in the following manner. Expanding x at $(k+1)^{th}$ iteration by Taylor series, about x^k and retaining terms upto first derivative

$$x^{k+1} = x^k + \dot{x}^k \Delta$$

Substituting the value of $\dot{x}$ from Eq. (2.1), we get,

$$x^{k+1} = x^k + \Delta [Ax^k + Bu^k] \tag{2.15}$$

Taking partial derivative of Eq. (2.15) with respect to a_{ij} and b_{ij}, it becomes,

$$\frac{\partial x^{k+1}}{\partial a_{ij}} = \frac{\partial x^k}{\partial a_{ij}} + \Delta \left[A \frac{\partial x^k}{\partial a_{ij}} + \frac{\partial A}{\partial a_{ij}} x^k \right]$$

$$\frac{\partial x^{k+1}}{\partial b_{ij}} = \frac{\partial x^k}{\partial b_{ij}} + \Delta \left[A \frac{\partial x^k}{\partial b_{ij}} + \frac{\partial B}{\partial b_{ij}} u^k \right] \tag{2.16}$$

The initial conditions are invariant with respect to a_{ij} and b_{ij}, i.e.

$$\frac{\partial x(o)}{\partial a_{ij}} = 0 \quad \text{and} \quad \frac{\partial x(o)}{\partial b_{ij}} = 0$$

The elements of the gradient x can now be determined by Eq. (2.16). An alternate procedure to determine the elements of gradient x, due to Taylor (9) is given in Appendix B.

Thus all the terms in Eq. (2.14) have been evaluated; $\nabla_c J$ can now be evaluated by Eq. (2.13).

The difference between the measured and computed responses (z-y), can be represented as quasi-linear with respect to a change in the unknown coefficients, i.e.

$$[z-y]_k \simeq [z-y]_{k-1} + \nabla_c [z-y]_k \Delta c_k$$

Using this approximation in the cost functional results in the following first and second gradients:

$$\nabla_c J = 2\left\{\int_0^T [(z-y)_k]^T D_1 [\nabla_c (z-y)]_k \, dt\right\}^T \quad (2.17)$$

$$\nabla_c^2 J = 2 \int_0^T [\{\nabla_c (z-y)\}_k]^T D_1 [\nabla_c (z-y)]_k \, dt + 2 \int_0^T [\{\nabla_c^2 (z-y)\}_k]^T D_1 (z-y)_k \, dt \quad (2.18)$$

The second term of $\nabla_c^2 J$ diminishes, as the response error $(z - y_k)$ diminishes. The modified Newton-Raphson technique as

developed by Taylor (9) neglects this term. Thus, the final relation for $\nabla_c^2 J$ becomes

$$\nabla_c^2 J = 2 \int_0^T [\{\nabla_c (z-y)\}_k]^T D_1 [\nabla_c (z-y)]_k \, dt \quad (2.19)$$

Now, substituting the values of $\nabla_c J$ and $\nabla_c^2 J$ in Eq. (2.12), the modified Newton-Raphson algorithm becomes

$$\Delta c_{k+1} = -\left\{\int_0^T \{[\nabla_c (z-y)]_k\}^T D_1 [\nabla_c (z-y)]_k \, dt\right\}^{-1} * \int_0^T \{[\nabla_c (z-y)]_k\}^T D_1 (z-y)_k \, dt \quad (2.20)$$

All the terms in Eq. (2.20) involve only the first gradient of (z-y). This greatly reduces the computation time and the approximation improves as the solution is approached.

For computational purposes, the integrals are treated as summations. In the indicial notations the Eq. (2.20) then becomes.

$$\Delta c = -\left\{\sum_{i=1}^{l} [\nabla_c (z^i-y^i)]^T D_1 \nabla_c (z^i-y^i)\right\}^{-1} * \sum_{j=1}^{l} [\nabla_c (z^j-y^j)^T D_1 (z^j-y^j)] \quad (2.21)$$

where subscripts i and j are the indices indicating the time sample, and l is the total number of samples.

Equation (2.21) is the final form of the modified Newton-Raphson algorithm.

2.4.3. Davidon-Fletcher-Powell (D.F.P.) Minimization Technique:

In the D.F.P. technique the local Hessian matrix $(\nabla_c^2 J)k$ is replaced by an approximate matrix H^k. The method of computing this matrix completely eliminates the need for evaluating second derivatives and performing matrix inversions, and yet the sequence of iterations converges quadratically to the minimum point (18).

After selecting a starting point, a direction of search is computed as follows (19):

$$M_i^k = \frac{-\sum_{j=1}^{m} H_{i,j} \left(\frac{\partial J}{\partial c_j}\right)}{\left[\sum_{l=1}^{m}\left\{\sum_{j=1}^{m} H_{l,j} \left(\frac{\partial J}{\partial c_j}\right)\right\}^2\right]^{1/2}} \qquad (2.22)$$

where i, j, l = 1, 2 ...., m, m being the number of unknowns c_i, and k is the iteration index. M_i are the direction vector components, $\frac{\partial J}{\partial c_j}$ are the gradient vector components and H_{ij} are the elements of a positive definite matrix (mxm), which is initially chosen to be identity matrix. It is evident from Eq. (2.22) that the initial direction of the search is

the path of steepest descent.

A one-dimensional search is conducted in the direction chosen by Eq. (2.22) until a minimum is located utilizing the relation,

$$c_i^{k+1} = c_i^k + \alpha^* M_i^k \tag{2.23}$$

where α^* is the step size in the direction of search.

Now a convergence check is made. If the convergence is achieved, the process is terminated, otherwise, the matrix H^{k+1} is calculated as follows:

$$H^{k+1} = H^k + M^k - L^k$$

where

$$M^k = \frac{\Delta c^k \, (\Delta c^k)^T}{(\Delta c^k)^T \, ((\Delta G)^k)}$$

$$L^k = \frac{H^k \, (\Delta G)^k \, \left[(\Delta G)^k\right]^T \, H^k}{\left|(\Delta G)^k\right|^T \, H^k \, (\Delta G)^k} \tag{2.24}$$

$$\Delta c^k = c^{k+1} - c^k$$

$$(\Delta G)^k = \left(\frac{\partial J}{\partial c}\right)^{k+1} - \left(\frac{\partial J}{J c}\right)^k$$

The value of updated matrix H is substituted in Eq. (2.22) and a new one-dimensional search is made. The

process is repeated till the convergence is achieved.

Eq. (2.22) to (2.24) is the D.F.P. algorithm (see [19]).

CHAPTER 3

COMPUTATIONAL DETAILS, RESULTS AND DISCUSSION

3.1 INTRODUCTION:

The flight test data for the lateral-directional mode of a light subsonic aircraft, flying at a speed of approximately 280 M.P.H., are obtained from Ref. 17. This data is used for extraction of lateral-directional stability and control derivatives by the modified Newton Raphson method and the Davidon-Fletcher-Powell method. The computational details are discussed first. The stability and control derivatives obtained by the two methods are compared. The aircraft time histories calculated by using the extracted derivatives are compared with the flight test data. The results are then discussed and the sources of errors are listed. Finally the conclusions are drawn and some suggestions are made for the further work.

3.2 COMPUTATIONAL DETAILS:

The flow diagram depicting the steps involved in the extraction of lateral-directional stability and control derivatives from flight data, is given in Figure 2. The listing of the programmes developed for the modified Newton Raphson method and for the D.F.P. method are given in Appendix C and D respectively. The different variables used for the computations are defined in the program listing.

The input to the programmes consist of the initial guesses of the stability and control derivatives, the control input state, and the aircraft measured responses p, r, β, ϕ, $\dot{p}$, $\dot{r}$ and $\dot{\beta}$. In absence of the specific information and for convenience, the weighting matrix D_1 is taken as identity matrix. Since the values of y_{δ_a} and y_{δ_r} are generally small, they are taken as zero (see e.g. 16, 21). The value of y_ϕ is calculated from the speed of the aircraft. This reduces the number of unknowns to 15 i.e. 8 in matrix A and 7 in matrix B. The responses are measured from 0 to 10 seconds at interval of 0.1 seconds. Thus there are 101 data points of aircraft responses. It is assumed that only the values of p, r, β and ϕ are known. The values of $\dot{p}$, $\dot{r}$, and $\dot{\beta}$ are calculated by differentiating p, r and β by finite difference method. The time histories input is given in Appendix E.

The Newton Raphson programme consists of the main programme and subroutines CALDIF and MATINV. The subroutine CALDIF calculates the response of the aircraft and ∇_c (z-y), denoted by CDX and H in the programme. The subroutine MATINV calculates the inverse of ∇_c^2 J. The increment in the derivative values is calculated in the main program and the values are updated at each iteration. The process is repeated till the minimum value of the cost functional J is achieved. The final values of aircraft responses are given as output in Appendix F.

The D.F.P. propgramme consists of the main programme and subroutines CALDIF, FUNCT and DFP. The subroutine CALDIF calculates the response of the aircraft and ∇_c (z-y). The subroutine FUNCT calculates the value of cost functional J and its gradient ∇_c J. The subroutine DFP first carries out a one dimensional search until a minima is located. If the convergence is achieved, the process is terminated, otherwise the value of the matrix H is updated and a new one dimensional search is made. The values of theparameters are updated at each iteration. The iterative process is continued till the minima is achieved. The final values of the aircraft responses are given as output in Appendix G.

3.3 RESULTS AND DISCUSSION:

The aircraft time histories calculated from the stability and control derivatives, extracted from the flight data, by the modified Newton Raphson method, are shown in Fig.3. The time histories obtained by the D.F.P. method are shown in Fig.4. It can be seen from these figures that in both the cases the aircraft time histories match the flight data well. The fit obtained by the D.F.P. method is slightly better than that obtained by the modified Newton Raphson method. It can also be noted that none of the responses have shown any divergence from the flight data. There is a slight variation at the peak values of some of the responses. The possible reasons for the same are enumerated in Section 3.4.

The values of the stability and control derivatives obtained by the two methods, are listed in Table 1. It is observed that the values of eight of the twelve derivatives extracted from the flight data by the two methods, are close to each other. The values of the 'damping-in-roll' derivative, L_p; the side force derivative due to side-slip, Y_β, and the yaw due to roll derivative, N_p are sufficiently in error. The sign of the roll due to yaw derivative, L_r is reversed. One of the reasons of this could be that these parameters are weak parameters, that cannot be accurately identified from flight test data (13).

It has been shown by Frederick (20) that stability derivatives L_β, N_r, N_β are most influential in determining lateral-directional stability. The table 1 shows that these derivatives have been recovered accurately by both the methods. It is also noted that even though some of the unimportant derivatives such as N_p, Y_β, L_r are sufficiently in error, the time histories are closely matched with the flight data in both the cases.

The modified Newton Raphson technique when used with zero initial values as starting values of the stability and control derivatives, gave large fit errors. An erroneous set of stability and control derivatives was obtained and the calculated responses differed considerably from the measured aircraft dynamics. It is, therefore, essential to provide

appropriate initial approximations to these derivatives which could be used as starting values in the matrices A and B.

The D.F.P. method was also found to be sensitive to the initial starting values of the stability and control derivatives.

The modified Newton Raphson technique first converged to a minimum value of the fit error and then diverged. However, the D.F.P. method never showed this tendency. The D.F.P. method always converged to the local minima. Thus the D.F.P. method can be satisfactorily used in cases where the modified Newton Raphson technique fails to converge. The convergence in case of the modified Newton Raphson method was faster than the D.F.P. method, resulting in less computational time.

Calculations for the problem considered are performed on IBM 7044 computer system. The order of magnitude of the computer time required by the two techniques for solving the present problem, is as follows:

Modified Newton Raphson technique 10-12 minutes

Davidon-Fletcher-Powell technique 35-40 minutes

3.4 SOURCE OF ERRORS:

The aircraft behaviour cannot be predicted very accurately by the linear dynamical model of the aircraft.

Similarly, the assumption of no coupling between the longitudinal and the lateral-directional modes, is not accurate enough for the flight conditions of the analysed data. Besides, this the factors that account for the difference in the computed and measured time histories of the aircraft and in the evaluation of stability and control derivatives, are as follows:

1. The input pulse amplitude is not kept small resulting in the violation of assumptions of small perturbations. Increasing amplitude will introduce inertial or aerodynamic nonlinearities.

2. Only the partial data was available for the analysis. The value of the response variables $\dot{p}$, $\dot{r}$ and $\dot{\beta}$ were calculated by differentiating p, r and β by the finite difference method. This introduces error in the estimation of $\dot{p}$, $\dot{r}$ and $\dot{\beta}$ and thus in the estimation of parameters and the aircraft time histories.

3. Since the results of all parameter identification procedures depend heavily on the quality of the test data available, there is a need for minimizing the instrumentation errors. Some of the more common instrument induced errors include random noise, calibration errors, mounting inaccuracies, instrument bias and time lag. Consequently, to achieve reliable results, the data must be conditioned by compensating for instrument shortcomings.

4. The inputs used for exciting the specific modes of an aircraft should be compatible with the derivatives to be extracted. For example, if one is interested in measuring $C_{L_{\delta_a}}$, he should perform a manoeuvre in which $C_{L_{\delta_a}}$ is a dominant factor, such as rapid roll.

5. The large variations in some of the parameters is due to the fact that these parameters are weak parameters and they cannot be accurately identified from flight test data (13). Though the values of parameters like N_p, Y_β, L_r vary sufficiently by the two methods but their effect on the time histories is not predominant.

3.5 CONCLUSIONS:

Both the techniques viz. the modified Newton Raphson method and the D.F.P. method give good results for the example considered for a light subsonic aircraft. Though the values of unimportant parameters like N_p, L_r, Y_β vary sufficiently, their effect on the time histories is not predominant. The calculated time histories of the aircraft match well with the flight data for both the methods.

Both the methods are found to be sensitive to the initial values of the derivatives.

Though the modified Newton Raphson method converges faster than the D.F.P. method, it shows a divergence after the

minima is achieved. However, the D.F.P. method never shows such behaviour. It always converges steadily to the minimum value. Thus the D.F.P. method could prove superior in cases where the Newton Raphson method does not show convergence.

3.6 SUGGESTIONS FOR FURTHER WORK:

1. An algorithm be evolved which would be insensitive to the initial estimates of the parameters and converges fast to the correct values of the parameters.
2. The modified Newton Raphson method and the D.F.P. method should be used to extract the stability and control derivatives of high performance aircraft and the wide bodied transport aircraft, to confirm that these methods could be applied to common configurations of aircraft.
3. An effort be made to find out a suitable weighting matrix D_1 to avoid divergence after converging to the minima in case of the modified Newton Raphson method.

   The effect of neglecting the second term in Eq. (2.18) be explored to find out its effect on the overall results of the modified Newton Raphson technique.
4. An effort be made to modify the D.F.P. method suitably to reduce the computation time.

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APPENDIX A

EQUATIONS OF MOTION:

The lateral-directional equations of motion of an airplane are (21):

$$-L_\beta \beta + \dot{p} - L_p p - \frac{I_{xz}}{I_{xx}} \dot{r} - L_r r = L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \tag{A.1}$$

$$-N_\beta \beta \frac{I_{xz}}{I_{zz}} \dot{p} - N_p p + \dot{r} - N_r r = N_{\delta_a} \delta_a + N_{\delta_r} \delta_r \tag{A.2}$$

$$\dot{\beta} - Y_\beta \beta - Y_p p - \frac{g}{V} \phi + r - Y_r r = Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r \tag{A.3}$$

Neglecting the product of inertia I_{xz} and assuming $y_r = 0$, these equations reduce to the following:

$$\dot{p} = L_p p + L_r r + L_\beta \beta + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r + L_o \tag{A.4}$$

$$\dot{r} = N_p p + N_r r + N_\beta \beta + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r + N_o \tag{A.5}$$

$$\dot{\beta} = Y_p p - r + Y_\beta \beta + Y_\phi \phi + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r + Y_o \tag{A.6}$$

where $Y_\phi = \frac{g}{V}$ and L_o, N_o, Y_o are the effect of uncertain bias on the measurement of $\dot{p}$, $\dot{r}$ and $\dot{\beta}$.

Realizing that $\dot{\phi} = p$. Eq. (A.4 - A.6) when written in the matrix form become

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix} \tag{A.7}$$

Equation (A.7) can be represented as

$$\dot{x}(t) = A\, x(t) + B\, u(t) \tag{A.8}$$

where

$$x = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} \quad u = \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix}$$

Note that x, $\dot{x}$ and u are time varying while the elements of matrices A and B are constant.

APPENDIX B

DETERMINATION OF ELEMENTS OF ∇_c x:

The elements of gradient x can be determined in the following manner.

Differentiating state Eq. (2.8) with respect to a_{ij}, we obtain

$$\frac{\partial \dot{x}}{\partial a_{ij}} = \frac{\partial A}{\partial a_{ij}} x + A \frac{\partial x}{\partial a_{ij}} + \cancelto{0}{\frac{\partial B}{\partial a_{ij}}} u + B \cancelto{0}{\frac{\partial u}{\partial a_{ij}}}$$

$$= A \frac{\partial x}{\partial a_{ij}} + \frac{\partial A}{\partial a_{ij}} x \qquad \text{(B.1)}$$

By solving the differential equation (B.1), we obtain for $\frac{\partial x}{\partial a_{ij}}$.

$$\frac{\partial x}{\partial a_{ij}} = \int_0^t e^{A(t-\tau)} \; A_{a_{ij}} \; x(\tau) \, d\tau$$

Again for coefficients in B, we obtain

$$\frac{\partial x}{\partial b_{ij}} = \int_0^t e^{B(t-\tau)} \; B_{b_{ij}} \; u(\tau) \, d\tau$$

Similarly if the change with respect to initial conditions $x_i(o)$ is considered, we obtain

$$\frac{\partial x}{\partial x_i(o)} = e^{At} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

where 1 appears in i^{th} row.

APPENDIX C

LISTING OF MODIFIED NEWTON-RAPHSON PROGRAMME

```
IBJOB
IBFTC   MAIN
                                   PROGRAMM  VARIABLES
     A   SYSTEM  MATRIX
     B   CONTROL  MATRIX
     X   MEASURED  RESPONSE
     U   CONTROL  DATA
   CDX   CALCULATED  RESPONSE
   ITR   NUMBER  OF  ITERATIONS
     E   STATE  TRANSFORMATION  MATRIX
     F   CONTROL  TRANSFORMATION  MATRIX
   DEL   TIME  INCREAMENT
    TI   RECORD  LENGTH
     N   NUMBER  OF  TIME  POINTS
     D   WEIGHTING  MATRIX  FOR  MEASURED  RESPONSE
     H   GRADIENT  OF  CALCULATED  RESPONSE  VARIABLE W.R.T.
       PARAMETER  VECTOR
  NOPS   NUMBER  OF  PROBLEMS  TO  BE  SOLVED
    DX   DERIVATIVE  OF  RESPONSE  VARIABLE
     Z   DERIVATIVE  OF  COST  FUNCTIONAL W.R.T.  PARAMETER  VECTOR
    AF   SECOND  DERIVATIVE  OF  COST  FUNCTIONAL W.R.T.
       PARAMETER  VECTOR
***THIS PROGRAM EVALUATES AIRCRAFT PARAMETERS FROM FLIGHT DATA
      DIMENSION X(110,7),U(110,3),A(4,4),B(4,3),DX(110,4),CDX(110,9),
    1AY(110),H(110,7,15),Z(110),AF(20,20),TX(20,1)
      DIMENSION D(7) ,F(30)
     COMMON/ANT/A,B,X,U
       CALL FLUN(30000)
*******************************************
      READ INPUT DATA
      READ100,NOPS
00    FORMAT(I2)
      CALL FLOW(30000)
      DO10NOP=1,NOPS
       READ1,TI,N
      FORMAT(F8.4,I3)
      DEL=TI/FLOAT(N-1)
     READ2,((X(I,J),J=1,4),(U(I,J),J=1,3),I=1,N)
     FORMAT(7F10.6)
      READ8,((A(I,J),J=1,4),I=1,4),((B(I,J),J=1,3),I=1,4)
      FORMAT(10F8.4)
      PRINT 700
00    FORMAT(10X,*MATRIX&A& IS *)
      PRINT 710,((A(I,J),J=1,4),I=1,4)
10    FORMAT(5X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4)
      PRINT 720
20    FORMAT(10X,*MATRIX &B& IS*)
      PRINT730,((B(I,J),J=1,3),I=1,4)
30    FORMAT(5X,F8.4,5X,F8.4,5X,F8.4)
*******************************************
      ITERATION LOOP STARTS
      ITR=1
*******************************************
      CALCULATE DERIVATIVES OF RESPONSE VARIABLES
      DO3J=1,N
      DO3K=1,4
      IF(J.GT.1)GOTO4
      DX(J,K)=(X(J+1,K)-X(J,K))/DEL
      GOTO3
       IF(J.EQ.N)GOTO5
      DX(J,K)=(X(J+1,K)-X(J-1,K))/DEL*0.5
     GOTO3
      DX(J,K)=(X(J,K)-X(J-1,K))/DEL
      CONTINUE
      DO 9 J=1,N
```

```
      DO 9 K=5,7
      X(J,K)=DX(J,K-4)
      PRINT750
)     FORMAT(/9X,100(1H*)/12X,*TIME*,1H*,*CONTROL    VARIABLES*,1H*,*R
     1E    S    P    O    N    S    E    V    A    R    I    A    B    L    E    S*/,
     2 9X,100(1H*)/12X,*SEC.*,2X,*AILERON*,1X,*RUDDER*,12X,*ROLL*,6X,*YA
     3W*,3X,*SIDE SLIP*,2X,*ROLL*,7X,*ROLL*,5X,*YAW*,5X,*SIDE SLIP*,/,19
     4X,*ANGLE*,5X,*ANGLE*,12X,*VEL.*,6X,*VEL*,4X,*ANGLE*,6X,*ANGLE*,6X,
     5*ACC.*,5X,*ACC.*,6X,*VEL.*//20X,*DEG.*,4X,*DEG.*,10X,*DEG/SEC*,3X,
     6*DEG/SEC*,4X,*DEG*,8X,*DEG.*,3X,*DEG/SEC*,3X,*DEG/SEC*,3X,*DEG/SEC
     7*/9X,100(1H*))
      TT=-DEL
      DO 550 I=1,N
      TT=TT+DEL
0     PRINT700,TT,(U(I,J),J=1,3),(X(I,J),J=1,7)
0     FORMAT(10X,F6.2,2F8.4,F6.3,7F10.6/)
**********************************************
      CALCULATE COST FUNCTIONALAND ITS DERIVATIVE DEL C J
9     CALLCALDIF(CDX,N,DEL,H)
     DO6 J=1,15
      F(ITR)=0.0
      Z(J)=0.
       DO6K=1,N
      DO6JK=1,7
  ZX=(Z-Y)    DIFFERENCE OF RESPONCE
      IF(JK.GT.4) GO TO 7
      ZX=X(K,JK)-CDX(K,JK)
      IF(K.EQ.1.OR.K.EQ.N) ZX=ZX/2.
      GO TO 6
    ZX=DX(K,JK-4)-CDX(K,JK)
    IF(K.EQ.1.OR.K.EQ.N) ZX=ZX/2.0
      GO TO 6
      F(ITR)=F(ITR)+ZX*ZX
      Z(J)=Z(J)+H(K,JK,J)*ZX
      PRINT 800,F(ITR)
0      FORMAT(5X,*VALUE OF FIT ERROR IS *,E15.7)
      IF(ITR.EQ.1) GOTO 600
      IF(F(ITR).GT.F(ITR-1)) GO TO 90
0     CONTINUE
**********************************************
      CALCULATE DEL SQ. C J
      DO10J=1,20
      DO10K=1,20
      AF(J,K)=0.
      DO11JK=1,N
      XNO=1.0
      IF(JK.EQ.1.OR.JK.EQ.N)XNO=0.5
     DO11 J=1,15
     DO11 K=1,15
      DO11KJ=1,7
      AF(J,K)=AF(J,K)+H(JK,KJ,J)*H(JK,KJ,K)*XNO
      DO200 J=1,15
      Z(J)=Z(J)*DEL
      DO200 K=1,15
0     AF(J,K)=AF(J,K)*DEL
**********************************************
      CALCULATE DELTA C
     DO 15 J=1,20
  15 TX(J,1)=0.0
      CALL MATINV(AF,15,TX,1)
     DO14 J=1,15
     TX(J,1)=0.0
     DO14 K=1,15
      TX(J,1)=AF(J,K)*Z(K)+TX(J,1)
**********************************************
```

```
   DC7K=1,4
  CDX(J,K)=CDX(J-1,K)+CDX(J-1,K+4) *DEL
  DC8 K=1,4
  CDX(J,K+4)=C.
  DC9JK=1,4
  CDX(J,K+4)=CDX(J,K+4)+C(K,JK)*CDX(J,JK)
  DC10JK=1,3
0 CDX(J,K+4)=CDX(J,K+4)+B(K,JK)*U(J,JK)
  CCNTINUE
**************************************************
  FORMULATE MATRICES E AND F
  DC15J=1,7
   DC15K=1,4
   IF(J.GT.4)GOTO16
   E(J,K)=C.
   IF(J.EQ.K)E(J,K)=1.
   GCTC15
   E(J,K)=A(J-4,K)
  CONTINUE
   DC17J=1,7
   DC17K=1,3
  IF(J.GT.4)GCTO18
  F(J,K)=0.
  GCTC17
  F(J,K)=B(J-4,K)
  CONTINUE
  CALCULATE DEL C (Z Y)
 DO19 J=1,15
  DC19K=1,N
  DO19JK=1,7
 H(K,JK,J)=0.
  IDETIFY ELEMENTS OF C IN MATRICES A AND B
 DO20 J=1,15
   J1=J
 IF(J.GT.8)J1=J-8
 IF(J.GT.8) GCTO21
   IJ=0
  IF(J.GT.3)IJ=1
  IF(J.GT.6)IJ=2
  I1=J-3*IJ
  IF((IJ.EQ.2).ANC.(I1.EQ.2))I1=3
  I2=IJ+1
  GOTC22
 IJ=1
  IF(J1.GT.3)IJ=2
  IF(J1.GT.6)IJ=3
  I1=J1-3*(IJ-1)
 IF(IJ.EQ.3)I1=3
  DO20K=1,N
  DC2CCJK=1,7
 IF(J.GT.8)GCTO23
 IF(JK.EQ. (4+I2))H(K,JK,J)=H(K,JK,J)-CDX(K,I1)
  GOTC 200
  IF(JK.EQ.(4+IJ) )H(K,JK,J)=H(K,JK,J)-U(K,I1)
  CONTINUE
 IF(K.EQ.1) GO TO 20
 DO 26 KJ1=1,4
 Z(KJ1)=0.C
  DC26 KJ=1,4
  Z(KJ1)=Z(KJ1)+A(KJ1,KJ)*H(K-1,KJ,J)
  DC 27 KJ=1,K
  Z(KJ)=Z(KJ)+DEL*H(K-1,KJ,J)
 IF(J.GT.8)GCTO25
  Z(I2)=Z(I2)+DEL*CDX(K-1,I1)
 GO TO 34
```

```
      Z(IJ)=Z(IJ)+DEL*U(K-1,I1)
34    DO 31 JK=1,7
      DO 31 KJ=1,4
31    H(K,JK,J)=H(K,JK,J) E(JK,KJ)*Z(KJ)
20    CONTINU
      RETURN
      END
```

```
FTC   SUB 2
      SUBROUTINE MATINV(A,N,P,LP)
      THIS SUBROUTINE FINDS INVERSE OF A MATRIX
    A IS INPUT MATRIX(N*N).ON RETURN IT GIVES INVERSE OF A
    P  IS MATRIX OF UNKNOWN VECTOR OF ORDER N*LP
    DIMENSION A(20,20),P(20,1),J1(20),J2(20),NX(20)
    DO 66 JI=1,N
    GRET=0.
    DO 44 M=1,N
     IF(JI.EQ.1) GO TO 15
     II=JI-1
    DO 34 M1=1,II
    JJ=J1(M1)
    IF(M.EQ.JJ) GO TO 44
     CONTINUE
     DO 45 K=1,N
     IF(JI.EQ.1) GO TO 79
    DO 33 K1=1,II
    JK=J2(K1)
    IF(K.EQ.JK) GO TO 45
    CONTINUE
    IF(ABS(A(M,K)).LT.ABS(GRET)) GO TO 45
    GRET=A(M,K)
    J1(JI)=M
    J2(JI)=K
     CONTINUE
     CONTINUE
    JA=J2(JI)
    JB=J1(JI)
    NLP=N+LP
    DO 22 M=1,N
    Y=A(M,JA)
    IF(M.EQ.JB) GO TO22
    DO 220 K=1,NLP
    IF(K.GT.N) GO TO 230
     A(M,K)=A(M,K)-A(JB,K)*Y/GRET
    GO TO 220
    KL=K-N
    P(M,KL)=P(M,KL)-P(JB,KL)*Y/GRET
    CONTINUE
    A(M,JA)=-Y/GRET
     CONTINUE
     DO 221 M=1,NLP
     IF(M.GT.N) GO TO225
    A(JB,M)=A(JB,M)/GRET
    GO TO 221
    ML=M-N
    P(JB,ML)=P(JB,ML)/GRET
     CONTINUE
    A(JB,JA)=1./GRET
     CONTINUE
     DO 800 KK=1,N
     DO 802 J=1,N
```

```
JK=J1(J)
JC=J2(J)
 IF(KK.EQ.JC) GO TO 804
 CONTINUE
NX(KK)=JK
 CONTINUE
IF(JK.LT.KK) JK=NX(JK)
 IF(JK.LT.KK) GO TO 806
 DO 808 JS=1,NLP
 IF(JS.GT.N) GO TO 810
Z=A(KK,JS)
A(KK,JS)=A(JK,JS)
 A(JK,JS)=Z
GO TO 808
KKL=JS-N
 Z=P(KK,KKL)
 P(KK,KKL)=P(JK,KKL)
P(JK,KKL)=Z
 CONTINUE
CONTINUE
DO 53 MN=1,N
DO 40 NN=1,N
JK=J1(NN)
 JC=J2(NN)
IF(MN.EQ.JK) GO TO 25
CONTINUE
 NX(MN)=JC
 CONTINUE
IF(JC.LT.MN) JC=NX(JC)
 IF(JC.LT.MN) GO TO 1
 DO 53 MM=1,N
  Z=A(MM,JK)
A(MM,JK)=A(MM,JC)
 A(MM,JC)=Z
 RETURN
 END
Y
```

APPENDIX D

G OF DAVIDON-FLETCHER-POWELL PROGRAMME

```
$JOB
$FTC    MAIN
C     DESCRIPTION OF PARAMETERS.
C       A    SYSTEM   MATRIX
C       B    CONTROL   MATRIX
C       X    MEASURED   RESPONSE
C       U    CONTROL   DATA
C     CDX    CALCULATED   RESPONSE
C     DEL    TIME   INCREAMENT
C      TI    RECORD   LENGTH
C       N    NUMBER   OF   TIME   POINTS
C       H    GRADIENT   OF   CALCULATED   RESPONSE   VARIABLE W.R.T.
C          PARAMETER   VECTOR
C    NOPS    NUMBER   OF   PROBLEMS   TO   BE   SOLVED
C      DX    DERIVATIVE   OF   RESPONSE   VARIABLE
C**THIS PROGRAM EVALUATES AIRCRAFT PARAMETERS FROM FLIGHT DATA
      DIMENSION X(110,7),U(110,3),A(4,4),B(4,3),DX(110,4) ,
     1CDX(110,8),C(15),G(15)
      COMMON A,B,X,U,N,DEL
      DATA EPS,LIMIT,NN/0.5,40,15/
      DATA EST/70./
      M=(NN*(NN+7))/2
      READ100,NOPS
0     FORMAT(I2)
      DO101NOP=1,NOPS
      READ1,TI,N
      FORMAT(F8.4,I3)
      DEL=TI/FLOAT(N-1)
      READ2,((X(I,J),J=1,4),(U(I,J),J=1,3),I=1,N)
      FORMAT(7F10.6)
      READ8,((A(I,J),J=1,4),I=1,4),((B(I,J),J=1,3),I=1,4)
      FORMAT(8F10.6)
      PRINT 700
0     FORMAT(10X,*MATRIX&A& IS *)
      PRINT 710,((A(I,J),J=1,4),I=1,4)
0     FORMAT(5X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4)
      PRINT 720
0     FORMAT(10X,*MATRIX &B& IS*)
      PRINT730,((B(I,J),J=1,3),I=1,4)
0     FORMAT(5X,F8.4,5X,F8.4,5X,F8.4)
      ITR=1
      DO3J=1,N
      DO3K=1,4
      IF(J.GT.1)GOTO4
      DX(J,K)=(X(J+1,K)-X(J,K))/DEL
      GOTO3
      IF(J.EQ.N)GOTO5
      DX(J,K)=(X(J+1,K)-X(J-1,K))/DEL*0.5
      GOTO3
      DX(J,K)=(X(J,K)-X(J-1,K))/DEL
      CONTINUE
      CALL FLUN(30000)
      PRINT750,((X(I,J),J=1,4),(DX(I,J),J=1,4),I=1,N)
0     FORMAT(5X,8(5X,F10.6)/)
      CALL FLOV(30000)
      DO 70 I=1,N
      DO 70 J=5,7
      X(I,J)=DX(I,J-4)
C     IDENTIFY UNKNOWN PARAMETERS IN MATRIX A AND B
      DO 60 I=1,15
      IF(I.GT.3)GOTO 10
      C(I)=A(1,I)
      GO TO 60
      IF(I.GT.6) GOTO 20
      C(I)=A(2,I-3)
```

```
 GC TC 60
 IF(I.GT.8) GOTO 30
 C(7)=A(3,1)
 C(8)=A(3,3)
 GC TC 60
 IF(I.GT.11) GO TO 40
 C(I)=B(1,I-8)
 GO TC 60
 IF(I.GT.14) GOTC 50
 C(I)=B(2,I-11)
 GO TC 60
 C(15)=B(3,3)
 CONTINUE
 PRINT300,(C(I),I=1,15)
 CALL FUNCT(NN,C,F,G)
 PRINT920,F
 FORMAT(5X,E14.6)
 CALL  DFP(NN,M,C,F,EST,EPS,LIMIT,IER,KOUNT)
 PRINT 300,(C(I),I=1,15)
  FORMAT(5X,8(2X,F12.6))
PRINT 310,IER,KOUNT,F
 FORMAT(5X,I3,7X,I2,7X,E14.6)
 CALL FUNCT(NN,C,F,G)
 PRINT 70C
 PRINT 710,((A(I,J),J=1,4),I=1,4)
 PRINT 72C
 PRINT 730,((B(I,J),J=1,3),I=1,4)
 CALL CALDIF(CDX,H)
 PRINT 999
 PRINT860
FORMAT(/9X,100(1H*)/12X,*TIME*,1H*,*CONTROL   VARIABLES*,1H*,*R
E  S  P  O  N  S  E  V  A  R  I  A  B  L  E  S*/,
 9X,100(1H*)/12X,*SEC.*,2X,*AILERON*,1X,*RUDDER*,12X,*ROLL*,6X,*YA
W*,4X,*SIDE SLIP*,2X,*ROLL*,7X,*ROLL*,5X,*YAW*,5X,*SIDE SLIP*,/,19
X,*ANGLE*,3X,*ANGLE*,12X,*VEL.*,6X,*VEL*,4X,*ANGLE*,6X,*ANGLE*,6X,
*ACC.*,5X,*ACC.*,6X,*VEL.*//20X,*DEG.*,4X,*DEG.*,10X,*DEG/SEC*,2X,
*DEG/SEC*,4X,*DEG*,8X,*DEG.*,3X,*DEG/SEC*,3X,*DEG/SEC*,3X,*DEG/SEC
*/9X,100(1H*))
 TT=-DEL
 DC 570 I=1,N
 TT=TT+DEL
 PRINT770,TT,(U(I,J),J=1,3),(CDX(I,J),J=1,7)
 FORMAT(9X,1H*,F6.2,2F8.4,F6.2,4F10.6,F11.6,2F10.6,1H*/)
 FORMAT(1H1)
 PRINT 999
 CONTINUE
 STOP
 END
```

```
  SUB 1

 SUBROUTINE  DFP(N,M,X,F,EST,EPS,LIMIT,IER,KOUNT)
N    =NUMBER OF INDEPENDENT VARIABLES
X    =INDEPENDENT VARIABLE VECTOR(INITIAL VALUES ON INPUT,
      OPTIMUM VALUES ON OUTPUT)
F    =FINAL MINIMUM VALUE OF THE OBJECTIVE FUNCTION
G    =FINAL GRADIENT VECTOR AT THE MINIMUM
H    =STORAGE VECTOR
M    =STORAGE VECTOR DIMENSION (N£(N+7))/2
EST    =ESTIMATE OF THE MINIMUM VALUE OF THE OBJECTIVE FUNCTION
EPS    =TEST VALUE REPRESENTIMG THE EXPECTED ABSOLUTE ERROR IN
```

```
         MOVEMENT
LIMIT  =MAXIMUM NUMBER OF ITERATIONS
IER    =ERROR PARAMETER
           IER=0 MEANS CONVERGENCE WAS OBTAINED
           IER=1 MEANS NO CONVERGENCE IN LIMIT ITERATIONS
           IER=-1 MEANS ERRORS IN GRADIENT CALCULATION
           IER=2 MEANS IT IS LIKELY THAT A MINIMUM DOES NOT EXIST
KOUNT  =ITERATION COUNTER

 DIMENSIONH(165),X(15),G(15),A(4,4),B(4,3),D(110,7),U(110,3)
 COMMON A,B,D,U,I1,DEL

   COMPUTE FUNCTION VALUE AND GRADIENT VECTOR FOR INITIAL ARGUMENT
 CALL FUNCT(N,X,F,G)

   RESET ITERATION COUNTER AND GENERATE IDENTITY MATRIX
IER=0
KOUNT=0
N2=N+N
N3=N2+N
N31=N3+1
 K=N31
DO 4  J=1,N
H(K)=1.
NJ=N-J
IF(NJ) 5,5,2
DO 3  L=1,NJ
KL=K+L
H(KL)=0.
K=KL+1

   START ITERATION LOOP
KOUNT=KOUNT+1
 KONT=KOUNT-1
 PRINT 900,KONT,(X(I),I=1,15)

   SAVE FUNCTION VALUE,ARGUMENT VECTOR AND GRADIENT VECTOR
 CALL FLUN(30000)
 CALL FLOV(30000)
OLDF=F
DO 9  J=1,N
K=N+J
H(K)=G(J)
K=K+N
H(K)=X(J)

   DETERMINE DIRECTION VECTOR H.
K=J+N3
T=0.
DO 8  L=1,N
T=T-G(L)*H(K)
IF(L-J) 6,7,7
K=K+N-L
GO TO 8
K=K+1
CONTINUE
H(J)=T

   CHECK WHETHER FUNCTION WILL DECREASE STEPPING ALONG H.
DY=0.
HNRM=0.
GNRM=0.

   CALCULATE DIRECTIONAL DERIVATIVE AND TEST VALUES FOR DIRECTION
   VECTOR H AND GRADIENT VECTOR G.
```

```
DO 10 J=1,N
HNRM=HNRM+ABS(H(J))
GNRM=GNRM+ABS(G(J))
DY=DY+H(J)*G(J)

   REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTIONAL
   DERIVATIVE APPEARS TO BE POSITIVE OR ZERO
IF(DY) 11,51,11

   REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTION
   VECTOR H IS SMALL COMPARED TO GRADIENT VECTOR G.
IF(HNRM/GNRM-EPS) 51,51,12

SEARCH MINIMUM ALONG DIRECTION H

   SEARCH ALONG H FOR POSITIVE DIRECTIONAL DERIVATIVE
FY=F
ALFA=2.*(EST-F)/DY
AMBDA=1.

   USE ESTIMATE FOR STEPSIZE ONLY IF IT IS POSITIVE AND LESS THAN
   1.OTHERWISE TAKE 1. AS STEP SIZE
IF(ALFA) 15,15,13
IF(ALFA-AMBDA) 14,15,15
AMBDA=ALFA
ALFA=0.

   SAVE FUNCTION AND DERIVATIVE VALUES FOR OLD ARGUMENT
FX=FY
DX=DY

   STEP ARGUMENT ALONG H
DO 17 I=1,N
X(I)=X(I)+AMBDA*H(I)

   COMPUTE FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT
 CALL FUNCT(N,X,F,G)
FY=F

   COMPUTE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT,TERMINATE
   SEARCH,IF DY IS POSITIVE.IF DY IS ZERO THE MINIMUM IS FOUND
DY=0.
DO 18 I=1,N
DY=DY+G(I)*H(I)
IF(DY) 19,36,22

   TERMINATE SEARCH ALSO IF THE FUNCTION VALUE INDICATES THAT
   A MINIMUM HAS BEEN PASSED
IF(FY-FX) 20,22,22

   REPEAT SEARCH AND DOUBLE STEPSIZE FOR FURTHER SEARCHES
AMBDA=AMBDA+ALFA
ALFA=AMBDA
   END OF SEARCH LOOP

   TERMINATE IF THE CHANGE IN ARGUMENT GETS VERY LARGE
IF(HNRM*AMBDA-1.E10) 16,16,21

   LINEAR SEARCH TECHNIQUE INDICATES THAT NO MINIMUM EXISTS
IER=2
RETURN

   INTERPOLATE CUBICALLY IN THE INTERVAL DEFINED BY THE SEARCH
   ABOVE AND COMPUTE THE ARGUMENT X FOR WHICH THE INTERPOLATION
   POLYNOMIAL IS MINIMIZED
```

```
T=0.
IF(AMBDA) 24,36,24
Z=3.*(FX-FY)/AMBDA+DX+DY
ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))
DALFA=Z/ALFA
DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
IF(DALFA) 51,25,25
W=ALFA*SQRT(DALFA)
ALFA=(DY+W-Z)*AMBDA/(DY+2.*W-DX)
DO 26 I=1,N
X(I)=X(I)+(T-ALFA)*H(I)

   TERMINATE,IF THE VALUE OF THE ACTUAL FUNCTION AT X IS LESS THAN
   THE FUNCTION VALUES AT  THE INTERVAL ENDS.OTHERWISE REDUCE
   THE INTERVAL BY CHOOSING ONE-END POINT EQUAL TO X AND REPEAT
   THE INTERPOLATION.WHICH END POINT IS CHOOSEN DEPENDS ON THE
   VALUE OF THE FUNCTION AND ITS GRADIENT AT X

 CALL FUNCT(N,X,F,G)
IF(F-FX)27,27,28
IF(F-FY) 36,36,28
DALFA=0.
DO 29 I=1,N
DALFA=DALFA+G(I)*H(I)
IF(DALFA) 30,33,33
IF(F-FX) 32,31,33
IF(DX-DALFA) 32,36,32
FX=F
DX=DALFA
T=ALFA
AMBDA=ALFA
GO TO 23
IF(FY-F) 35,34,35
IF(DY-DALFA) 35,36,35
FY=F
DY=DALFA
AMBDA=AMBDA-ALFA
GO TO 22

   COMPUTE DIFFERENCE VECTORS OF ATGUMENT AND GRADIENT FROM
   TWO CONSECUTIVE ITERATIONS
DO 37 J=1,N
K=N+J
H(K)=G(J)-H(K)
K=N+K
H(K)=X(J)-H(K)
 IF(KOUNT.EQ.1)GOTO 60
 IF(ABS(OLDF-F).LE.EP)GOTO55
 CONTINUE

   TERMINATE,IF FUNCTION HAS NOT DECREASED DURING LAST ITERATION
IF(OLDF-F+EPS) 51,38,38

   TEST LENGTH OF ARGUMENT DIFFERENCE VECTOR AND DIRECTION VECTOR
   IF AT LEAST N ITERATIONS HAVE BEEN EXECUTED.TERMINATE,IF
   BOTH ARE LESS THAN EPS
IER=0
IF(KOUNT-N) 42,39,39
T=0.
Z=0.
DO 40 J=1,N
K=N+J
W=H(K)
K=K+N
T=T+ABS(H(K))
```

```
Z=Z+W*H(K)
IF(HNRM-EPS) 41,41,42
IF(1-EPS) 56,56,42

   TERMINATE,IF NUMBER OF ITERATIONS WOULD EXCEED LIMIT
IF(KOUNT-LIMIT) 43,50,50

   PREPARE UPDATING OF MATRIX H
ALFA=0.
DO 47 J=1,N
K=J+N3
W=0.
DO 46 L=1,N
KL=N+L
W=W+H(KL)*H(K)
IF(L-J) 44,45,45
K=K+N-L
GO TO 46
K=K+1
CONTINUE
K=N+J
ALFA=ALFA+W*H(K)
H(J)=W

   REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF RESULTS
   ARE NOT SATISFACTORY
IF(Z*ALFA) 48,1,48

   UPDATE MATRIX H
K=N31
DO 49 L=1,N
KL=N2+L
DO 49 J=L,N
NJ=N2+J
H(K)=H(K)+H(KL)*H(NJ)/Z-H(L)*H(J)/ALFA
K=K+1
 PRINT900, KOUNT,(X(I),I=1,15)
 FORMAT(5X,I3,7(2X,F12.6))
 PRINT910,F
 FORMAT(2X,*VALUE OF  F  IS*,E14.6)
GO TO 5
   END OF ITERATION LOOP

   NO CONVERGENCE AFTER LIMIT ITERATIONS
IER=1
RETURN

   RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS
DO 52 J=1,N
K=N2+J
X(J)=H(K)
 CALL FUNCT(N,X,F,G)

   REPEAT IN DIRECTION OF STEEPEST DESCENT IF DERIVATIVE
   FAILS TO  BE SUFFICIENTLY SMALL
IF(GNRM-EPS) 55,55,53

   TEST FOR REPEATED FAILURE OF ITERATION
IF(IER) 56,54,54
IER=-1
GO TO 1
IER=0
RETURN
END
```

TC SUB 2

```
      SUBROUTINE FUNCT(ID,C,F,G)
C       ARGUMENT LIST
C            ID  = NO. OF INDEPENDET VARIABLES
C            C   =VECTOR OF X VALUES
C            F   =OBJECTIVE FUNCTION EQUATION.
C            G   =VECTOR OF OBJECTIVE FUNCTION DERIVATIVES,(N LONG).
      COMMONA,B,X,U,N,DEL
      DIMENSION C(15),G(15),A(4,4),B(4,3),X(110,7),U(110,3),CDX(110,8),
     1H(110,7,15)
      DO30 I=1,3
      DO 30 J=1,3
      IF(I.GT.1) GOTO 10
      A(I,J)=C(J)
      B(I,J)=C(J+8)
      GO TO 30
      IF(I.GT.2) GOTO20
      A(I,J)=C(J+3)
      B(I,J)=C(J+11)
      GO TO 30
      A(3,1)=C(7)
      A(3,3)=C(8)
      B(3,3)=C(15)
      CONTINUE
      CALL CALDIF(CDX,H)
      DO 6 J=1,15
      G(J)=0.0
      F=0.0
      DO 6 K=1,N
      DO 6 JK=1,7
      ZX=X(K,JK)-CDX(K,JK)
      IF(K.EQ.1.OR.K.EQ.N) ZX=ZX/2.
      ZX=ZX*10.E-02
      F=F+ZX*ZX
      G(J)=G(J)+H(K,JK,J)*ZX
      RETURN
      END
```

TC SUB 3

```
      SUBROUTINE CALDIF(CDX,H)
C     SUBROUTINE CALDIF CALCULATES RESPONSE AND  GRADIENT OF RESPONSE
C     W.R.T. UNKNOWN PARAMETERS

      COMMONA,B,X,U,N,DEL
      DIMENSION C(4,4),E(7,4),F(7,4),H(110,7,15),X(110,7),U(110,3),
     1A(4,4),B(4,3),CDX(110,8),Z(110)
      DO 300 I=1,4
      DO 300 J=1,4
      C(I,J)=A(I,J)
      DO1J=1,4
      CDX(1,J)=X(1,J)
      DO2J=1,4
      CDX(1,J+4)=0.
      DO4K=1,4
      CDX(1,J+4)=CDX(1,J+4)+C(J,K)*CDX(1,K)
      DO5K=1,3
      CDX(1,J+4)=CDX(1,J+4)+B(J,K)*U(1,K)
```

```
      CONTINUE
      DO6J=2,N
       DO7K=1,4
      CDX(J,K)=CDX(J-1,K)+CDX(J-1,K+4) *DEL
      DO8 K=1,4
      CDX(J,K+4)=0.
      DO9JK=1,4
      CDX(J,K+4)=CDX(J,K+4)+C(K,JK)*CDX(J,JK)
      DO10JK=1,3
      CDX(J,K+4)=CDX(J,K+4)+B(K,JK)*U(J,JK)
      CONTINUE
      DO15J=1,7
       DO15K=1,4
       IF(J.GT.4)GOTO16
       E(J,K)=0.
       IF(J.EQ.K)E(J,K)=1.
       GOTO15
       E(J,K)=A(J-4,K)
      CONTINUE
       DO17J=1,7
       DO17K=1,3
      IF(J.GT.4)GOTO18
      F(J,K)=0.
      GOTO17
      F(J,K)=B(J-4,K)
      CONTINUE
     DO19 J=1,15
      DO19K=1,N
      DO19JK=1,7
     H(K,JK,J)=0.
     DO20 J=1,15
       J1=J
     IF(J.GT.8)J1=J-8
     IF(J.GT.8) GOTO21
       IJ=0
      IF(J.GT.3)IJ=1
      IF(J.GT.6)IJ=2
      I1=J-3*IJ
      IF((IJ.EQ.2).AND.(I1.EQ.2))I1=3
      I2=IJ+1
      GOTO22
     IJ=1
      IF(J1.GT.3)IJ=2
      IF(J1.GT.6)IJ=3
      I1=J1-3*(IJ-1)
     IF(IJ.EQ.3)I1=3
      DO20K=1,N
      DO200JK=1,7
     IF(J.GT.8)GOTO23
     IF(JK.EQ. (4+I2))H(K,JK,J)=H(K,JK,J)-CDX(K,I1)
      GOTO 200
      IF(JK.EQ.(4+IJ) )H(K,JK,J)=H(K,JK,J)-U(K,I1)
      CONTINUE
     IF(K.EQ.1) GO TO 20
     DO 26 KJ1=1,4
     Z(KJ1)=0.0
      DO26 KJ=1,4
      Z(KJ1)=Z(KJ1)+A(KJ1,KJ)*H(K-1,KJ,J)
      DO 27 KJ=1,K
      Z(KJ)=Z(KJ)*DEL+H(K-1,KJ,J)
     IF(J.GT.8)GOTO25
      Z(I2)=Z(I2)+DEL*CDX(K-1,I1)
     GO TO 34
      Z(IJ)=Z(IJ)+DEL*U(K-1,I1)
34   DO 31 JK=1,7
```

```
      DO 31  KJ=1,4
   31 H(K,JK,J)=H(K,JK,J) E(JK,KJ)*Z(KJ)
   20 CONTINUE
      RETURN
      END
ENTRY
```

TIME SEC.	CONTROL VARIABLES AILERON ANGLE DEG.	RUDDER ANGLE DEG.		RESPONSE VARIABLES ROLL VEL. DEG/SEC	YAW VEL DEG/SEC	SIDE SLIP ANGLE DEG	ROLL ANGLE DEG.	ROLL ACC. DEG/SEC^2	YAW ACC. DEG/SEC^2	SIDE SLIP VEL. DEG/SEC
0.00	0.0000	0.0000	1.00	0.000090	-0.000042	0.000023	0.000002	0.001920	0.000750	0.000650
0.10	0.0000	0.0000	1.00	0.000222	-0.000117	0.000074	0.000002	0.001000	-0.000680	0.000545
0.20	0.0000	0.0000	1.00	0.000290	-0.000178	0.000132	0.000040	0.000330	-0.000540	0.000600
0.30	0.0000	0.0000	1.00	0.000288	-0.000225	0.000194	0.000070	-0.000375	-0.000400	0.000630
0.40	0.0000	0.0000	1.00	0.000215	-0.000258	0.000258	0.000100	-0.001080	-0.000255	0.000635
0.50	0.0000	0.0000	1.00	0.000072	-0.000276	0.000321	0.000114	-0.001760	-0.000110	0.000620
0.60	0.0000	0.0000	1.00	-0.000137	-0.000280	0.000382	0.000112	-0.002395	0.000020	0.000595
0.70	0.0000	0.0000	1.00	-0.000407	-0.000272	0.000440	0.000085	0.255185	-0.233600	0.005340
0.80	0.0000	0.0000	1.00	0.050900	-0.047000	0.001450	0.001000	7.000535	-2.543640	0.131200
0.90	4.9000	1.0000	1.00	1.399700	-0.509000	0.026680	0.040950	42.644499	-8.829500	0.879750
1.00	9.5500	1.0000	1.00	8.579800	-1.812900	0.177400	0.513200	65.841499	-12.142000	2.467100
1.10	2.5000	1.0000	1.00	14.568000	-2.937400	0.520100	1.720700	15.221000	-4.915500	3.725500
1.20	-4.3950	1.0000	1.00	11.624000	-2.796000	0.922500	3.077200	-38.175499	2.779500	3.664500
1.30	-4.4000	1.0000	1.00	6.932900	-2.381500	1.253000	4.007600	-48.147500	4.445000	2.863500
1.40	-4.4700	1.0000	1.00	1.994500	-1.907000	1.495200	4.455500	-47.682500	4.608500	1.553500
1.50	-2.2000	1.0000	1.00	-2.603600	-1.459800	1.643700	4.410800	-32.138500	2.694500	1.192500
1.60	0.3000	0.8100	1.00	-4.493200	-1.368100	1.733700	4.045300	-16.852000	1.290500	0.754500
1.70	0.3000	0.4250	1.00	-5.974000	-1.201700	1.794600	3.526400	-16.208500	2.402500	0.391500
1.80	0.3000	0.0000	1.00	-7.674900	-0.887600	1.812000	2.845000	-17.495000	3.808500	-0.118000

*	1.90	0.1000	-0.3400	1.00	-9.473000	-0.440000	1.771000	1.988000	-18.080500	5.061400	-0.766000*
*	2.00	0.1000	-0.7250	1.00	-11.291000	0.124680	1.658800	0.949500	-17.855000	6.137000	-1.524000*
*	2.10	0.1000	-1.1100	1.00	-13.044000	0.787400	1.466200	-0.268300	-16.635000	6.519100	[illegible]
*	2.20	0.1000	[illegible]	1.00	-14.618000	1.510500	1.186800	-1.653800	-13.790000	[illegible]	[illegible]
*	2.30	0.1000	[illegible]	1.00	-15.802000	2.167000	0.824700	-3.178500	-9.425000	[illegible]	[illegible]
*	2.40	0.1000	[illegible]	1.00	-16.503000	2.724500	0.394100	-4.758000	-4.325000	[illegible]	[illegible]
*	2.50	0.1000	[illegible]	1.00	-16.667000	3.172000	-0.089000	-6.461200	1.220000	[illegible]	[illegible]
*	2.60	0.1000	[illegible]	1.00	-16.259000	3.502400	-0.607300	-8.112300	7.005000	[illegible]	[illegible]
*	2.70	0.1000	[illegible]	1.00	-15.288000	3.712800	-1.143000	-9.693400	12.820000	[illegible]	-5.354500*
*	2.80	0.1000	[illegible]	1.00	-13.695000	3.803400	-1.678200	-11.146000	18.465000	0.333000	-5.262000*
*	2.90	0.1000	[illegible]	1.00	-11.573000	3.779200	-2.195400	-12.414000	23.748000	-0.776000	-5.001000*
*	3.00	0.1000	[illegible]	1.00	-8.945400	3.648200	-2.678400	-13.444000	28.455000	-1.788500	-4.584000*
*	3.10	0.1000	[illegible]	1.00	-5.873200	3.421500	-3.112200	-14.188000	32.568500	-2.674500	-4.028000*
*	3.20	0.1000	[illegible]	1.00	-2.431700	3.113300	-3.484000	-14.606000	35.829000	-3.409000	-3.354000*
*	3.30	0.1000	[illegible]	1.00	1.292600	2.739700	-3.783000	-14.665000	38.183500	-3.973500	-2.586000*
*	3.40	0.1000	[illegible]	1.00	5.205000	2.318600	-4.001200	-14.341000	39.566500	-4.352500	-1.751000*
*	3.50	0.1000	[illegible]	1.00	9.205900	1.869200	-4.133200	-13.621000	39.945000	-4.540000	-0.875500*
*	3.60	0.1000	[illegible]	1.00	13.194000	1.410600	-4.176300	-12.501000	39.320500	-4.534000	0.011000*
*	3.70	0.1000	[illegible]	1.00	17.070000	0.962400	-4.131000	-10.986000	37.725000	-4.338500	0.881500*
*	3.80	0.1000	[illegible]	1.00	20.739000	0.542900	-4.000000	-9.093500	35.215000	-3.965000	1.710000*
*	3.90	0.1000	[illegible]	1.00	24.113000	0.169400	-3.789000	-6.848100	31.880000	-3.428000	2.471000*
*	4.00	0.1000	[illegible]	1.00	27.115000	-0.142700	-3.505800	-4.283300	27.835001	-2.749000	3.142500*
*	4.10	0.1000	[illegible]	1.00	29.680000	-0.380400	-3.160500	-1.439700	23.205000	-1.951500	3.706000*
	4.20	0.1000	[illegible]	1.00	31.756000	-0.533000	-2.764600	1.636300	18.135000	-1.063000	4.147000*
	4.30	0.1000	[illegible]	1.00	33.307000	-0.593000	-2.331100	4.894000	12.795000	-0.113000	4.453000*
	4.40	0.1000	[illegible]	1.00	34.315000	-0.555600	-1.873600	8.279700	7.335000	0.868500	4.624000*
	4.50	0.1000	[illegible]	1.00	34.774000	-0.419300	-1.406300	11.739000	1.914999	1.849500	4.652000*
	4.60	0.1000	[illegible]	1.00	34.698000	-0.185700	-0.943200	15.217000	-3.305001	2.801000	4.541500*

*	4.80	9.1000	-1.2000	1.00	33.062000	0.553000	-0.083200	22.024000	-12.874999	4.503000	3.537500	*
*	4.90	9.1000	-1.2000	1.00	31.598000	1.041500	0.289500	25.260000	-18.879999	[illegible]	3.457500	*
*	5.00	[illegible]	-1.2000	1.00	29.284000	1.663700	0.608300	28.322000	-41.715000	8.869000	[illegible]	*
*	5.10	7.2500	[illegible]	1.00	23.255000	2.815300	0.821100	30.981000	-77.090000	[illegible]	[illegible]	*
*	5.20	9.5000	-0.9000	1.00	13.868000	4.408600	0.841200	32.849000	-90.435000	[illegible]	[illegible]	*
*	5.30	7.9000	[illegible]	1.00	5.128000	5.720500	0.648600	33.783000	-74.268750	[illegible]	[illegible]	*
*	5.40	[illegible]	[illegible]	1.00	-0.985750	6.625300	0.295800	33.967000	-46.652500	[illegible]	4.088000	*
*	5.50	2.3000	[illegible]	1.00	-4.202500	7.107800	-0.153000	33.685000	-17.734250	2.696500	4.637000	*
*	5.60	0.5000	-0.5000	1.00	-4.532600	7.163400	-0.631600	33.225000	10.697500	-1.529000	-4.607500	*
*	5.70	3.3000	-0.5000	1.00	-2.063000	6.802000	-1.074500	32.674000	37.964000	-5.582000	-3.535500	*
*	5.80	4.2000	-0.5000	1.00	3.060200	6.047000	-1.418700	32.904000	63.365000	-5.337000	-2.654000	*
*	5.90	6.9000	-0.5000	1.00	10.610000	4.934600	-1.605300	33.570000	84.943999	-12.490000	-0.820000	*
*	6.00	10.4000	-0.5000	1.00	20.049000	3.549000	-1.582700	35.095000	94.404999	-13.854500	1.379500	*
*	6.10	10.6000	-0.5000	1.00	29.491000	2.163700	-1.329400	37.576000	90.530001	-13.239000	3.641500	*
*	6.20	9.5000	-0.5000	1.00	38.155000	0.901200	-0.854400	40.972000	73.795000	-10.705500	5.662000	*
*	6.30	7.3000	-0.5000	1.00	44.250000	0.022600	-0.197000	45.115000	46.320000	-6.563000	7.155000	*
*	6.40	4.7000	-0.5000	1.00	47.419000	-0.411400	0.576600	49.722000	17.010000	-2.078500	7.570000	*
*	6.50	2.3000	-0.5000	1.00	47.652000	-0.393100	1.397000	54.499000	-12.005000	5.574500	8.092000	*
*	6.60	-0.1000	-0.5000	1.00	45.018000	0.703500	2.195000	59.155000	-39.955001	6.752500	7.532000	*
*	6.70	-2.5000	-0.4000	1.00	39.661000	0.957400	2.903400	63.410000	-64.175000	6.822000	6.339500	*
*	6.80	-3.7000	0.8000	1.00	32.183000	2.067900	3.462900	67.009000	-72.654998	7.115500	4.556000	*
*	6.90	-3.7000	[illegible]	1.00	25.130000	2.380500	3.894600	69.868000	-66.840000	-1.022000	4.149500	*
*	7.00	-3.7000	5.0900	1.00	18.815000	1.863500	4.292800	72.063000	-63.905001	-5.304000	4.015500	*
*	7.10	-3.7000	5.0700	1.00	12.349000	1.319700	4.697700	73.623000	-66.096500	-4.917500	4.005000	*
*	7.20	-3.7000	5.0500	1.00	5.595700	0.880000	5.093800	74.523000	-68.837500	-3.906000	3.832000	*
*	7.30	-3.7000	[illegible]	1.00	-1.418500	0.538500	5.464100	74.734000	-71.215500	-2.970000	3.494500	*
*	7.40	-3.7000	[illegible]	1.00	-8.647400	0.286000	5.792700	74.232000	-73.057499	-2.144000	3.007500	*

*	7.50	-3.7000	[illegible]	1.00	-16.030000	0.109700	6.065600	73.000000	-74.217555	-1.454000	2.391500	*
*	7.60	-3.7000	4.5100	1.00	-23.491000	-0.004800	6.271000	71.023000	-74.594995	-0.921000	1.470500	*
*	7.70	-3.7000	4.5[illegible]00	1.00	-30.549000	-0.074500	6.399700	68.301000	-74.095555	-0.563[illegible]00	0.871000	*
*	7.80	-3.7000	4.5100	1.00	-38.111000	-0.117500	6.445200	64.837000	-72.680001	-0.35[illegible]500	0.020000	*
*	7.90	-3.7000	4.5[illegible]00	1.00	-45.485000	-0.152800	6.403700	60.645000	-71.100001	0.788750	0.867500	*
*	8.00	-3.7000	[illegible]	1.00	-52.531000	-0.059750	6.271700	55.746000	-73.794999	[illegible]	2.[illegible]	*
*	8.10	-3.7000	0.4000	1.00	-60.244000	0.966600	5.991400	50.113000	-78.884957	14.[illegible]750	4.[illegible]	*
*	8.20	-3.4410	-1.0255	1.00	-68.308000	2.847600	5.459000	43.681000	-74.399998	18.115000	-6.731000	*
*	8.30	-2.9100	-0.9100	1.00	-75.124000	4.585600	4.645200	36.497000	-59.570003	15.701500	-9.[illegible]21500	*
*	8.40	-2.4140	[illegible]	1.00	-80.222000	5.987900	3.594700	28.714000	-41.460004	12.075500	11.418000	*
*	8.50	-1.9800	[illegible]	1.00	-83.416000	7.004700	2.361600	20.516000	-21.620002	8.136500	12.553000	*
*	8.60	-1.2900	[illegible]	1.00	-84.586000	7.615200	1.004100	12.099000	-1.334956	4.015000	13.894600	*
*	8.70	-0.2700	-0.4000	1.00	-82.683000	7.807700	-0.417320	3.668700	19.280005	-0.156500	14.230500	*
*	8.80	-0.3600	[illegible]	1.00	-80.730000	7.583900	-1.842000	-4.568600	39.064999	-4.185000	13.968400	*
*	8.90	-0.0955	[illegible]	1.00	-75.870000	6.970700	-3.211000	-12.412000	55.695000	-7.582000	13.166500	*
*	9.00	-0.0864	[illegible]	1.00	-69.591000	6.067500	-4.475300	-19.696000	68.710000	-10.258000	-11.928500	*
*	9.10	-0.0773	[illegible]	1.00	-62.128000	4.919100	-5.596700	-26.291000	79.474998	-12.511000	-10.338000	*
*	9.20	-0.0682	-0.1364	1.00	-53.696000	3.565300	-6.542900	-32.089000	87.974998	-14.342000	-8.454500	*
*	9.30	-0.0591	[illegible]	1.00	-44.533000	2.050700	-7.287600	-37.006000	93.990002	-15.705000	-6.344500	*
*	9.40	-0.0500	-0.1000	1.00	-34.898000	0.424300	-7.811800	-40.980000	97.365000	-16.567500	-4.079500	*
*	9.50	-0.0410	-0.0820	1.00	-25.060000	-1.262800	-8.103500	-43.978000	98.044998	-16.915500	-1.733000	*
*	9.60	-0.0320	-0.0640	1.00	-15.289000	-2.958800	-8.158400	-45.994000	96.057500	-16.750500	0.620000	*
*	9.70	-0.2270	-0.0[illegible]	1.00	-5.848500	-4.612900	-7.979500	-47.047000	91.509500	-16.090000	2.907000	*
*	9.80	-0.0140	-0.0279	1.00	3.012500	-6.176800	-7.577000	-47.183000	84.587500	-14.966500	5.058000	*
*	9.90	-0.0045	-0.0091	1.00	11.069000	-7.606200	-6.967900	-46.472000	58.600500	-10.405500	4.921500	*
*	10.00	0.0000	0.0000	1.00	14.793000	-8.257900	-6.592700	-45.825000	36.640000	-6.517000	3.752000	*

TIME	CONTROL VARIABLES			RESPONSE VARIABLES						
SEC.	AILERON ANGLE	RUDDER ANGLE		ROLL VEL.	YAW VEL	SIDE SLIP ANGLE	ROLL ANGLE	ROLL ACC.	YAW ACC.	SIDE SLIP VEL.
	DEG.	DEG.		DEG/SEC	DEG/SEC	DEG	DEG.	DEG/SEC^2	DEG/SEC^2	DEG/SEC
0.00	0.0000	0.0000	1.00	0.000090	-0.000042	0.000023	0.000007	1.429174	[illegible]	[illegible]
0.10	0.0000	0.0000	1.00	0.143007	-0.027676	0.017366	0.000013	[illegible]	[illegible]	[illegible]
0.20	0.0000	0.0000	1.00	0.259344	-0.050664	0.037421	0.014312	0.867223	[illegible]	[illegible]
0.30	0.0000	0.0000	1.00	0.346067	-0.068420	0.059425	0.040246	0.551664	[illegible]	[illegible]
0.40	0.0000	0.0000	1.00	0.401233	-0.080156	0.082565	0.074853	0.228060	[illegible]	[illegible]
0.50	0.0000	0.0000	1.00	0.424039	-0.085895	0.106009	0.114976	-0.092182	[illegible]	[illegible]
0.60	0.0000	0.0000	1.00	0.414821	-0.085673	0.128927	0.157380	-0.398028	[illegible]	0.228640
0.70	0.0000	0.0000	1.00	0.375018	-0.079729	0.150531	0.198862	-0.679164	[illegible]	[illegible]
0.80	0.0000	-0.5000	1.00	0.307100	-0.068498	0.170094	0.236364	0.815847	-1.614637	0.188826
0.90	4.9000	1.0000	1.00	0.388684	-0.229961	0.186977	0.267074	41.970648	-8.820854	0.327559
1.00	9.5500	1.0000	1.00	4.585749	-1.112047	0.219733	0.305942	78.196748	13.799882	1.514910
1.10	2.9000	[illegible]	1.00	12.405424	-2.492035	0.371224	0.764517	17.327442	-5.248905	3.432708
1.20	-4.3930	[illegible]	1.00	14.138168	-3.016925	0.714494	2.006060	-42.927388	3.377914	3.956406
1.30	-4.4000	[illegible]	1.00	9.845429	-2.679134	1.110135	3.418876	-47.735218	4.575819	3.129775
1.40	-4.4700	1.0000	1.00	5.071907	-2.241652	1.423112	4.403419	-51.347050	5.152267	2.188432
1.50	-2.2000	[illegible]	1.00	-0.062798	-1.726525	1.641956	4.910610	-34.683549	3.051249	1.167759
1.60	0.1000	0.5100	1.00	-3.531152	-1.421200	1.758731	4.904330	-17.464260	1.375016	0.514767
1.70	0.1000	0.4250	1.00	-5.277578	-1.283699	1.810208	4.551215	-19.069133	2.842603	0.179464
1.80	0.1000	0.0400	1.00	-7.184492	-0.999439	1.828155	4.023457	-20.204005	4.200341	-0.306023

*	1.90	0.1000	[illegible]	1.00	-9.204892	-0.579404	1.797552	3.305008	-20.659945	5.394972	-0.918147	*
*	2.00	0.1000	[illegible]	1.00	-11.270887	-0.035907	1.705738	2.384519	-20.316525	[illegible]	1.627[illegible]48	*
*	2.10	0.1000	[illegible]	1.00	-13.302539	0.603423	1.542973	1.237430	-19.043599	[illegible]	[illegible]	*
*	2.20	0.1000	-1.3000	1.00	-15.206899	1.331036	1.302526	-0.072624	-16.089435	[illegible]	[illegible]	*
*	2.30	0.1000	-1.3000	1.00	-16.815842	2.052832	0.980037	-1.592514	-11.450547	[illegible]	[illegible]	*
*	2.40	0.1000	-1.3000	1.00	-17.960897	2.681155	0.583009	-3.275098	5.911974	[illegible]	[illegible]	*
*	2.50	0.1000	-1.3000	1.00	-18.552094	3.199048	0.127981	-5.071184	0.260624	[illegible]	[illegible]	*
*	2.60	0.1000	-1.3000	1.00	-18.524031	3.594043	-0.366691	-6.926397	6.865432	[illegible]	[illegible]	*
*	2.70	0.1000	-1.3000	1.00	-17.837488	3.858492	-0.881704	-8.778800	13.573968	[illegible]	[illegible]	*
*	2.80	0.1000	-1.3000	1.00	-16.480091	3.989708	-1.397512	-10.562549	20.140559	0.002309	-4.974575	*
*	2.90	0.1000	-1.3000	1.00	-14.466031	3.989959	-1.895009	-12.210558	26.311740	-1.237757	-4.611844	*
*	3.00	0.1000	-1.3000	1.00	-11.834857	3.866163	-2.356193	-13.657161	31.854440	-2.364341	-4.085764	*
*	3.10	0.1000	-1.3000	1.00	-8.649413	3.629729	-2.764770	-14.840647	36.564045	-3.338820	-3.419130	*
*	3.20	0.1000	-1.3000	1.00	-4.593009	3.295847	-3.106683	-15.705588	40.270707	-4.128875	-2.638761	*
*	3.30	0.1000	-1.3000	1.00	-0.963938	2.882959	-3.370559	-16.204889	42.844522	-4.709487	-1.774857	*
*	3.40	0.1000	-1.3000	1.00	3.318514	2.412011	-3.548044	-16.301482	44.199162	-5.063639	-0.859864	*
*	3.50	0.1000	-1.3000	1.00	7.738430	1.905647	-3.634031	-15.989631	44.293897	-5.182735	0.072710	*
*	3.60	0.1000	-1.3000	1.00	12.167819	1.387379	-3.626760	-15.195788	43.133971	-5.066708	0.989496	*
*	3.70	0.1000	-1.3000	1.00	16.481216	0.880702	-3.527810	-13.979006	40.769365	-4.723828	1.858430	*
*	3.80	0.1000	-1.3000	1.00	20.558153	0.408320	-3.341967	-12.330884	37.292014	-4.170226	2.649846	*
*	3.90	0.1000	-1.3000	1.00	24.287354	-0.008703	-3.076983	-10.275069	32.831619	-3.429146	3.337468	*
*	4.00	0.1000	-1.3000	1.00	27.570516	-0.351617	-2.743236	-7.846333	27.550228	-2.529968	3.859251	*
*	4.10	0.1000	-1.3000	1.00	30.325538	-0.604614	-2.353311	-5.089282	21.635804	-1.507041	4.318061	*
*	4.20	0.1000	-1.3000	1.00	32.489119	-0.755318	-1.921504	-2.056728	15.295030	-0.398361	4.582[illegible]69	*
*	4.30	0.1000	-1.3000	1.00	34.018621	-0.795155	-1.463288	1.192184	8.745614	0.755845	4.685537	*
*	4.40	0.1000	-1.3000	1.00	34.893183	-0.719570	-0.994734	4.594046	2.208384	1.914582	4.627509	*
*	4.50	0.1000	-1.3000	1.00	35.114021	-0.528112	-0.531943	8.083364	-4.100552	3.017546	4.414[illegible]86	*
*	4.60	0.1000	-1.3000	1.00	34.703965	-0.224357	-0.090474	11.594766	-9.977304	4.086523	4.056615	*

*	4.70	0.1000	-1.3000	1.00	33.706234	0.184295	0.315187	15.065163	-15.237107	[illegible]	3.569289*
*	4.80	0.1000	-1.3000	1.00	32.182524	0.686964	0.672116	18.435786	-19.720393	[illegible]	2.972[illegible]96*
*	4.90	0.1000	-1.3000	1.00	30.210484	1.269740	0.969365	21.654038	-23.297855	[illegible]	[illegible]
*	5.00	[illegible]	[illegible]	1.00	27.880699	1.916233	1.198308	24.675086	-45.692369	[illegible]	[illegible]
*	5.10	-7.7000	[illegible]	1.00	23.311462	2.883563	1.352888	27.463156	-86.376824	[illegible]	[illegible]
*	5.20	-9.5000	-0.9000	1.00	14.673779	4.421727	1.386940	29.794302	-101.649683	[illegible]	[illegible]
*	5.30	-7.9000	-0.5000	1.00	4.508810	6.051027	1.216467	31.261680	-82.650145	[illegible]	[illegible]
*	5.40	-5.3000	-0.5000	1.00	-3.756204	7.285935	0.827638	31.712561	-52.831007	[illegible]	-5.462569*
*	5.50	2.3000	0.5000	1.00	-9.039305	8.086249	0.281381	31.336940	-21.597034	[illegible]	-6.330104*
*	5.60	0.5000	-0.5000	1.00	-11.199008	8.420729	-0.351629	30.453010	10.069710	-1.443627	-6.473162*
*	5.70	5.3000	-0.5000	1.00	-10.192037	8.276366	-0.998945	29.313109	41.185852	-6.178778	-5.899522*
*	5.80	6.1000	-0.5000	1.00	-6.073452	7.658489	-1.588937	28.293905	70.801567	-10.682683	-4.643607*
*	5.90	8.9000	-0.5000	1.00	1.006705	6.590220	-2.053298	27.686559	98.033466	-14.788979	-2.760754*
*	6.00	10.4000	-0.5000	1.00	10.810051	5.111323	-2.329374	27.787230	111.578719	-16.888440	0.328714*
*	6.10	10.6000	-0.5000	1.00	21.967923	3.422479	-2.362245	28.868235	111.020909	-16.881219	2.330238*
*	6.20	9.5000	-0.5000	1.00	33.070014	1.734357	-2.129221	31.065027	96.386186	-14.749813	4.891701*
*	6.30	7.1000	-0.5000	1.00	42.708632	0.259375	-1.640051	34.372028	68.135985	-10.558078	7.040622*
*	6.40	4.7000	-0.5000	1.00	49.522231	-0.796433	-0.935989	38.642891	37.660174	-5.907426	8.482174*
*	6.50	2.3000	-0.5000	1.00	53.288248	-1.387175	-0.087771	43.595114	5.960310	-0.983011	9.178590*
*	6.60	-0.1000	-0.5000	1.00	53.884278	-1.485476	0.830128	48.923939	-25.934790	4.024087	9.120739*
*	6.70	-2.5000	-0.5000	1.00	51.290799	-1.083067	1.742201	54.312366	-57.006020	8.923720	8.323885*
*	6.80	-3.7000	0.9000	1.00	45.590197	-0.190695	2.574590	59.441446	-71.694534	7.219138	6.830516*
*	6.90	-3.7000	3.7000	1.00	38.420743	0.531216	3.257642	64.000465	-69.385673	-1.145000	5.450455*
*	7.00	-3.7000	[illegible]	1.00	31.482176	0.416716	3.802687	67.842539	-70.067245	-4.704430	4.961766*
*	7.10	-3.7000	[illegible]	1.00	24.475451	-0.053725	4.298864	70.990756	-74.933564	-3.321446	4.802[illegible]79*
*	7.20	-3.7000	[illegible]	1.00	16.982095	-0.385869	4.779081	73.438301	-79.480906	-2.003406	4.422227*
*	7.30	-3.7000	[illegible]	1.00	9.034004	-0.586210	5.221304	75.136510	-83.420177	-0.804183	3.841988*
*	7.40	-3.7000	[illegible]	1.00	0.691987	-0.666628	5.605503	76.039910	-86.495913	0.228369	3.088052*

* 7.60	-3.7000	4.9700	1.00	16.803546	-0.341948	6.133552	75.313348	-89.249652	1.641466	1.155202*
* 7.70	-3.7000	4.9300	1.00	25.728515	-0.377801	6.253073	73.632993	-88.653215	[illegible]	0.127445*
* 7.80	-3.7000	4.9100	1.00	34.593636	-0.180924	6.265817	[illegible]	-90.641247	[illegible]	[illegible]
* 7.90	-3.7000	4.9100	1.00	43.257961	0.021437	6.169180	67.600757	-93.211075	[illegible]	[illegible]
* 8.00	-3.7000	3.4400	1.00	51.579068	0.201887	5.964596	63.274961	-83.606445	[illegible]	[illegible]
* 8.10	-3.7000	3.4400	1.00	59.939713	0.862469	5.657406	58.117054	-87.676542	[illegible]	[illegible]
* 8.20	-3.4430	[illegible]	1.00	68.727366	2.497664	5.199684	52.123082	-82.650845	[illegible]	-7.0[illegible]
* 8.30	-2.9100	[illegible]	1.00	76.990451	4.468918	4.494291	45.250345	-66.727645	10.[illegible]	-9.727[illegible]
* 8.40	2.4140	-0.7790	1.00	83.663213	6.125711	3.521556	37.551300	-47.549403	12.791[illegible]	-11.847112*
* 8.50	1.9000	-0.4500	1.00	88.418155	7.404872	2.336845	29.184979	-25.980591	8.554321	-13.349750*
* 8.60	1.3900	-0.5[illegible]	1.00	91.016213	8.261304	1.001870	20.343163	-2.914990	4.043715	-14.198450*
* 8.70	-0.8700	-0.4000	1.00	91.307712	8.665675	-0.417975	11.241542	20.819319	-0.588776	-14.379681*
* 8.80	-0.3600	-0.2543	1.00	89.225780	8.606797	-1.855943	2.110771	44.226593	-5.222375	-13.904196*
* 8.90	-0.0955	-0.1510	1.00	84.803120	8.084560	-3.246363	-6.811807	64.258567	-9.155021	-12.800261*
* 9.00	-0.0864	-0.1780	1.00	78.377262	7.169058	-4.526389	-15.292119	80.091139	-12.256602	-11.188255*
* 9.10	-0.0773	-0.1250	1.00	70.368148	5.943397	-5.645214	-23.129845	93.395301	-14.898445	-9.202428*
* 9.20	-0.0682	-0.1164	1.00	61.028618	4.453558	-6.565457	-30.166660	103.728622	-16.997745	-6.921182*
* 9.30	-0.0591	-0.1[illegible]80	1.00	50.855756	2.753778	-7.257575	-36.269521	110.759356	-18.437846	-4.431291*
* 9.40	-0.0500	-0.1000	1.00	39.579620	0.904994	-7.700705	-41.335096	114.282069	-19.327123	-1.825215*
* 9.50	-0.0410	-0.0820	1.00	28.151613	-1.027718	-7.883226	-45.293078	114.221596	-19.499835	0.802509*
* 9.60	-0.0320	-0.0640	1.00	16.729453	-2.977702	-7.802975	-48.108239	110.631822	-19.012953	3.358835*
* 9.70	-0.0270	-0.0455	1.00	-5.666271	-4.878997	-7.467092	-49.781184	102.041237	-17.659116	5.755321*
* 9.80	-0.0140	-0.0273	1.00	4.537852	-6.645909	-6.891559	-50.347811	93.728262	-16.208593	7.875633*
* 9.90	-0.0046	-0.0091	1.00	13.910679	-8.266768	-6.103996	-49.894026	81.109976	-14.016691	9.722217*
* 10.00	0.0000	0.0000	1.00	22.021676	-9.668437	-5.131774	-48.502958	66.228209	-11.365974	11.200677*

TIME*CONTROL VARIABLES*R E S P O N S E V A R I A B L E S

SEC.	AILERON ANGLE DEG.	RUDDER ANGLE DEG.		ROLL VEL. DEG/SEC	YAW VEL DEG/SEC	SIDE SLIP ANGLE DEG	ROLL ANGLE DEG.	ROLL ACC. DEG/SEC^2	YAW ACC. DEG/SEC^2	[illegible]
* 0.00	0.0000	0.0000	1.00	0.000090	-0.000042	0.000023	0.000002	0.003002	-0.00085	[illegible]
* 0.10	0.0000	0.0000	1.00	0.000390	-0.000101	-0.000233	0.000011	0.005886	[illegible]	[illegible]
* 0.20	0.0000	0.0000	1.00	0.000979	-0.000220	-0.000472	0.000050	0.008402	-0.0017[illegible]	[illegible]
* 0.30	0.0000	0.0000	1.00	0.001819	-0.000395	-0.000685	0.000148	0.010479	-0.0021[illegible]	[illegible]
* 0.40	0.0000	0.0000	1.00	0.002867	-0.000612	-0.000866	0.000330	0.012061	-0.002527	[illegible]
* 0.50	0.0000	0.0000	1.00	0.004073	-0.000865	-0.001010	0.000616	0.013110	-0.002788	[illegible]
* 0.60	0.0000	0.0000	1.00	0.005384	-0.001144	-0.001113	0.001024	0.013607	-0.002927	[illegible]
* 0.70	0.0000	0.0000	1.00	0.006745	-0.001437	-0.001172	0.001562	0.013551	-0.002953	[illegible]
* 0.80	0.0000	0.5000	1.00	0.008100	-0.001732	-0.001185	0.002237	2.496644	-1.755949	[illegible]
* 0.90	4.9000	1.0000	1.00	0.257764	-0.177327	-0.001153	0.003047	50.766604	-8.450521	[illegible]
* 1.00	9.5500	1.0000	1.00	5.334424	-1.022379	0.018197	0.028823	91.383483	12.756375	[illegible]
* 1.10	2.5000	1.0000	1.00	14.472773	-2.298017	0.157192	0.562265	18.701929	-4.601991	[illegible]
* 1.20	-4.5930	1.0000	1.00	16.342965	-2.758216	0.486984	2.009543	-50.349448	3.339867	[illegible]
* 1.30	-4.4000	1.0000	1.00	11.208021	-2.424229	0.875118	3.643839	-52.968653	4.093202	[illegible]
* 1.40	-4.4700	1.0000	1.00	6.011155	-2.014909	1.192990	4.774641	-54.630392	4.650504	[illegible]
* 1.50	-2.2000	1.0000	1.00	0.548116	-1.549858	1.429861	5.375757	-33.287885	2.607240	[illegible]
* 1.60	0.1000	0.6100	1.00	-2.780672	-1.289134	1.577816	5.430568	-12.677335	1.108605	[illegible]
* 1.70	0.1000	0.4280	1.00	-4.048406	-1.178274	1.671264	5.152501	-15.034544	2.614485	[illegible]
* 1.80	0.1000	0.0400	1.00	-5.551860	-0.916825	1.739063	4.747661	-16.935094	4.034756	[illegible]

*	1.90	0.1000	-0.2400	1.00	-7.245369	-0.535350	1.764449	4.152474	-18.275957	[illegible]	[illegible]	
*	2.00	0.1000	-0.7200	1.00	-9.065365	0.010140	1.732471	3.407938	-18.671466	[illegible]	[illegible]	
*	2.10	0.1000	-1.1100	1.00	-10.531111	0.644082	1.630646	2.561601	-18.295140	[illegible]	[illegible]	
*	2.20	0.1000	-1.2000	1.00	-12.760647	1.366732	1.448935	1.468490	-17.994034	[illegible]	[illegible]	
*	2.30	0.1000	-1.3000	1.00	-14.360081	2.088294	1.180225	0.152425	-11.830815	[illegible]	[illegible]	
*	2.40	0.1000	-1.3000	1.00	-15.543182	2.721723	0.826024	-1.243582	-6.908215	[illegible]	[illegible]	
*	2.50	0.1000	-1.3000	1.00	-16.233483	3.290113	0.405905	-2.797699	-1.415545	[illegible]	[illegible]	
*	2.60	0.1000	-1.3000	1.00	-16.375038	3.660554	-0.070505	-4.421247	4.414557	[illegible]	[illegible]	*
*	2.70	0.1000	-1.3000	1.00	-15.951542	3.944468	-0.584098	-6.058751	10.365221	[illegible]	[illegible]	*
*	2.80	0.1000	-1.3000	1.00	-14.897020	4.097796	-1.116848	-7.652105	16.213789	[illegible]	[illegible]	*
*	2.90	0.1000	-1.3000	1.00	-13.275641	4.121033	-1.650462	-9.141807	21.748057	-1.029256	[illegible]	*
*	3.00	0.1000	-1.3000	1.00	-11.100835	4.015107	-2.166574	-10.469371	26.771093	-2.179847	-4.823620	*
*	3.10	0.1000	-1.3000	1.00	-8.423726	3.801122	-2.649536	-11.579454	31.107750	-3.211557	-4.326289	*
*	3.20	0.1000	-1.3000	1.00	-5.312947	3.479962	-3.081965	-12.421827	34.610178	-4.061807	-3.652649	*
*	3.30	0.1000	-1.3000	1.00	-1.851929	3.071782	-3.451230	-12.953122	37.161745	-4.765860	-2.945549	*
*	3.40	0.1000	-1.3000	1.00	1.864245	2.595396	-3.745865	-13.138314	38.680617	-5.237973	-2.114297	*
*	3.50	0.1000	-1.3000	1.00	5.732307	2.071598	-3.957295	-12.951890	39.121543	-5.491715	-1.225708	*
*	3.60	0.1000	-1.3000	1.00	9.664461	1.522427	-4.079866	-12.378659	38.476619	-5.520268	-0.311687	*
*	3.70	0.1000	-1.3000	1.00	13.492123	0.970400	-4.110974	-11.414213	36.774770	-5.326430	0.598802	*
*	3.80	0.1000	-1.3000	1.00	17.169600	0.437757	-4.051094	-10.065000	34.080025	-4.920366	1.473564	*
*	3.90	0.1000	-1.3000	1.00	20.577602	-0.054280	-3.903698	-8.348040	30.488666	-4.319123	[illegible]	*
*	4.00	0.1000	-1.3000	1.00	23.626469	-0.486192	-3.675083	-6.290280	26.125358	-3.545926	[illegible]	*
*	4.10	0.1000	-1.3000	1.00	26.239005	-0.840785	-3.374108	-3.927633	21.138435	-2.629261	[illegible]	*
*	4.20	0.1000	-1.3000	1.00	28.352848	-1.103713	-3.011847	-1.303733	15.694485	-1.601923	[illegible]	*
*	4.30	0.1000	-1.3000	1.00	29.922297	-1.263905	-2.601182	1.531552	9.972447	-0.499648	[illegible]	*
*	4.40	0.1000	-1.3000	1.00	30.919541	-1.313870	-2.156337	4.523782	4.157415	0.639938	[illegible]	*
*	4.50	0.1000	-1.3000	1.00	31.335283	-1.249876	-1.692282	7.615736	-1.565640	1.778700	[illegible]	*
*	4.60	0.1000	-1.3000	1.00	31.178718	-1.072006	-1.224720	10.749264	-7.018007	2.879252	[illegible]	*

*	4.70	0.1000	-1.3000	1.00	30.476918	-0.784081	-0.768567	13.867136	-12.032070	[illegible]	[illegible]*
*	4.80	0.1000	-1.3000	1.00	29.273412	-0.392483	-0.338451	16.514627	[illegible]	[illegible]	[illegible]
*	4.90	0.1000	-1.3000	1.00	27.627462	0.085255	0.052237	19.842188	[illegible]	[illegible]	[illegible]
*	5.00	-2.3500	-1.3000	1.00	25.809152	0.656659	0.391762	[illegible]	[illegible]	[illegible]	[illegible]
*	5.10	-7.2500	-1.3000	1.00	21.604810	1.524536	0.670063	[illegible]	[illegible]	[illegible]	[illegible]
*	5.20	-9.5000	-0.5000	1.00	11.325127	2.926590	0.836576	27.266400	[illegible]	[illegible]	[illegible]
*	5.30	-7.9000	-0.5000	1.00	0.885986	4.396087	0.811837	28.458573	-85.742576	[illegible]	[illegible]
*	5.40	-5.1000	-0.5000	1.00	-7.738271	5.479516	0.572111	28.522571	-52.049170	[illegible]	[illegible]
*	5.50	2.5000	-0.5000	1.00	-12.947088	6.162110	0.173375	27.748744	-18.248074	[illegible]	[illegible]
*	5.60	8.5000	-0.5000	1.00	-14.767398	6.426698	-0.321487	26.454485	14.885445	-1.547567	[illegible]
*	6.70	9.3000	-0.5000	1.00	-12.278851	6.271939	-0.847481	24.977746	46.616600	-5.597344	[illegible]
*	5.80	6.1000	-0.5000	1.00	-8.617171	5.712205	-1.339687	23.649860	76.247239	-9.352806	-3.557128*
*	5.90	8.9000	-0.5000	1.00	-0.992447	4.776924	-1.735400	22.788143	103.143076	-12.674847	-2.467563*
*	6.00	10.4000	-0.5000	1.00	9.321860	3.509440	-1.976156	22.688898	114.565392	-14.116275	-0.334216*
*	6.10	10.6000	-0.5000	1.00	20.778399	2.097812	-2.009578	23.621084	110.716789	-13.649714	1.967574*
*	6.20	9.5000	-0.5000	1.00	31.850878	0.732840	-1.812780	25.698924	92.141850	-11.319912	4.201479*
*	6.30	7.1000	-0.5000	1.00	41.064213	-0.399151	-1.392632	28.883932	59.708844	-7.291487	6.076057*
*	6.40	4.7000	-0.5000	1.00	47.035097	-1.123300	-0.785027	32.990353	26.778891	-2.918921	7.318181*
*	6.50	2.3000	-0.5000	1.00	49.712986	-1.413132	-0.053209	37.693862	-5.864980	1.487165	7.657755*
*	6.60	-0.1000	-0.5000	1.00	49.126488	-1.266415	0.736567	42.665160	-37.446639	5.811814	7.807346*
*	6.70	-2.5000	-0.5000	1.00	45.381824	-0.685234	1.517281	47.577809	-67.223745	9.897180	7.061001*
*	6.80	-3.7000	[illegible]	1.00	38.659449	0.304485	2.222381	52.115991	-76.305041	7.464028	5.695320*
*	6.90	-3.7000	[illegible]	1.00	31.028944	1.050887	2.792913	55.981936	-65.141171	-1.414640	4.506789*
*	7.00	-3.7000	[illegible]	1.00	24.514827	0.909423	3.243592	59.084830	-60.318460	-5.399412	4.268410*
*	7.10	-3.7000	[illegible]	1.00	18.482981	0.369482	3.670433	61.536312	-62.544044	-4.391234	4.419824*
*	7.20	-3.7000	5.0000	1.00	12.228577	-0.069641	4.112416	63.384610	-64.815755	-3.377011	4.401[illegible]*
*	7.30	-3.7000	5.0000	1.00	5.747001	-0.407342	4.552574	64.607468	-66.929091	-2.398641	4.217294*
*	7.40	-3.7000	5.0000	1.00	-0.545908	-0.647206	4.974303	65.182167	-68.695122	-1.495422	3.876022*

*	7.50	-3.7000	5.0000	1.00	-7.815420	-0.796749	5.361906	65.087576	-69.898466	0.7[illegible]	[illegible]	[illegible]
*	7.60	-3.7000	4.9700	1.00	14.805088	-0.870538	5.701093	64.316093	-70.345725	[illegible]	[illegible]	[illegible]
*	7.70	-3.7000	4.9500	1.00	21.659447	-0.875585	5.979923	67.875520	-70.379239	[illegible]	[illegible]	[illegible]
*	7.80	-3.7000	4.9500	1.00	28.896867	-0.832005	6.187493	60.6[illegible]9582	-69.357047	[illegible]	[illegible]	[illegible]
*	7.90	-3.7000	4.9[illegible]00	1.00	35.832572	-0.758242	6.316430	57.749395	-67.441771	[illegible]	[illegible]	[illegible]
*	8.00	-3.7000	5.4000	1.00	42.577348	-0.673886	6.361658	54.166637	-72.039472	5.[illegible]	[illegible]	[illegible]
*	8.10	-3.7000	0.4600	1.00	49.781195	-0.676542	6.321210	49.908903	-82.689192	18.9[illegible]	[illegible]	[illegible]
*	8.20	-3.4430	-1.0355	1.00	58.048115	1.521939	6.138301	44.930783	-80.675564	19.[illegible]	[illegible]	[illegible]
*	8.30	-2.9500	-0.9100	1.00	66.115670	3.485578	5.704991	39.125971	-65.566965	16.86320	7.[illegible]	
*	8.40	-2.4140	-0.7750	1.00	72.672366	5.171899	4.988929	32.514404	-47.864160	13.495800	9.365487*	
*	8.50	-1.9000	-0.6500	1.00	77.458181	6.521459	4.032580	25.247160	-28.299326	9.690854	-11.467872*	
*	8.60	-1.3960	-0.5210	1.00	80.288713	7.490544	2.885795	17.501289	-7.576763	5.583077	-12.821103*	
*	8.70	-0.8700	-0.4000	1.00	81.046583	8.048851	1.603643	9.472418	13.668312	1.332229	-13.556581*	
*	8.80	-0.3608	-0.2643	1.00	79.619151	8.182075	0.244548	1.367760	34.641519	-2.572451	-13.763333*	
*	8.90	-0.0935	-0.1910	1.00	76.215599	7.884830	-1.131805	-6.600215	52.051570	-6.674349	-13.241163*	
*	9.00	-0.0884	-0.1330	1.00	71.010442	7.217395	-2.465922	-14.221775	65.326224	-9.669491	-12.414662*	
*	9.10	-0.0772	-0.1250	1.00	64.477818	6.250446	-3.707388	-21.322819	76.759974	-12.319676	-11.088712*	
*	9.20	-0.0682	-0.1164	1.00	56.801821	5.018458	-4.816259	-27.770601	85.980950	-14.544415	-9.419082*	
*	9.30	-0.0591	-0.1180	1.00	48.203726	3.564017	-5.757765	-33.450789	92.692566	-16.273045	-7.459710*	
*	9.40	-0.0500	-0.1000	1.00	38.934469	1.936712	-6.503336	-38.271155	96.691582	-17.455684	-5.261294*	
*	9.50	-0.0410	-0.0820	1.00	29.265311	0.191144	-7.031455	-42.164601	97.872182	-18.063445	-2.967649*	
*	9.60	-0.0320	-0.0640	1.00	19.478093	-1.615201	-7.328240	-45.091132	96.223326	-18.086020	-0.594951*	
*	9.70	-0.2270	-0.0455	1.00	9.855260	-3.423803	-7.387735	-47.038943	89.925563	-17.226858	1.757846*	
*	9.80	-0.0140	-0.0273	1.00	-0.869004	-5.156469	-7.211951	-48.024497	84.984916	-16.446011	3.979116*	
*	9.90	-0.0045	-0.0091	1.00	7.635487	-6.801070	-6.814039	-48.110797	75.760478	-14.852137	6.068185*	
*	10.00	0.0000	0.0000	1.00	15.211535	-8.286284	-6.207421	-47.347249	64.461075	-12.780473	7.918722*	

TABLE 1

COMPARISON OF STABILITY AND CONTROL DERIVATIVES

Derivative	Value obtained by modified N.R.method	Value obtained by D.F.P.method	Value obtained by Ref.17
L_p	-0.2575	-0.5088	-0.2740
L_r	-0.1513	0.2078	0.0628
L_β	-13.4469	-11.9081	-11.4000
N_p	+0.0015	0.0380	0.0013
N_r	-0.1760	-0.2242	-0.2081
N_β	2.4986	2.4513	2.1701
Y_p	0.0773	0.0702	0.0083
Y_β	-0.6788	-0.3709	-0.3150
L_{δ_a}	8.0890	9.3776	8.2232
L_{δ_r}	3.4846	4.9674	4.1772
N_{δ_a}	-1.1231	-1.0185	-1.1406
N_{δ_r}	-3.5475	-3.5062	-3.7614

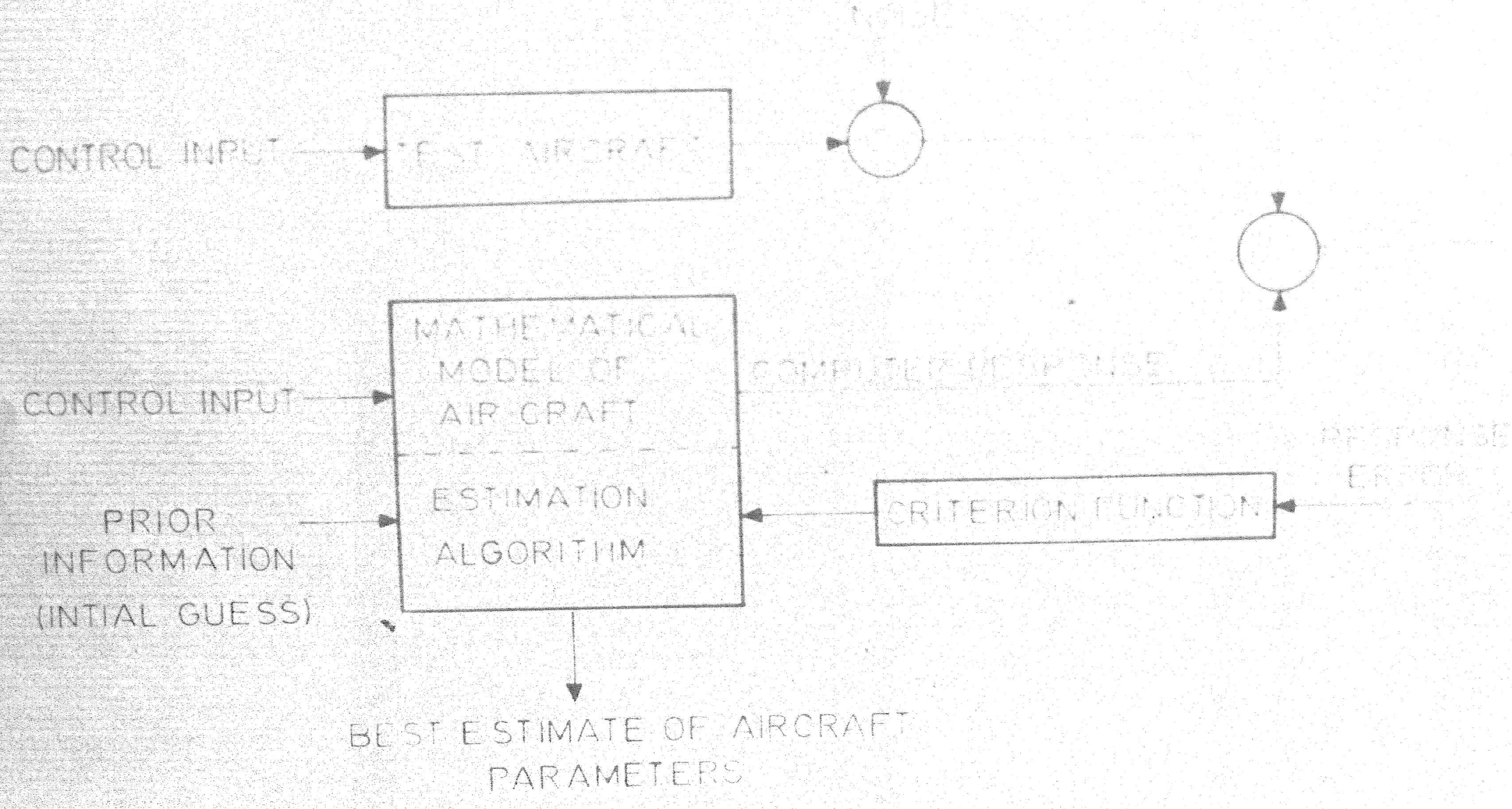


FIG 1 BASIC CONCEPT OF AIRCRAFT PARAMETER ESTIMATION TECHNIQUE

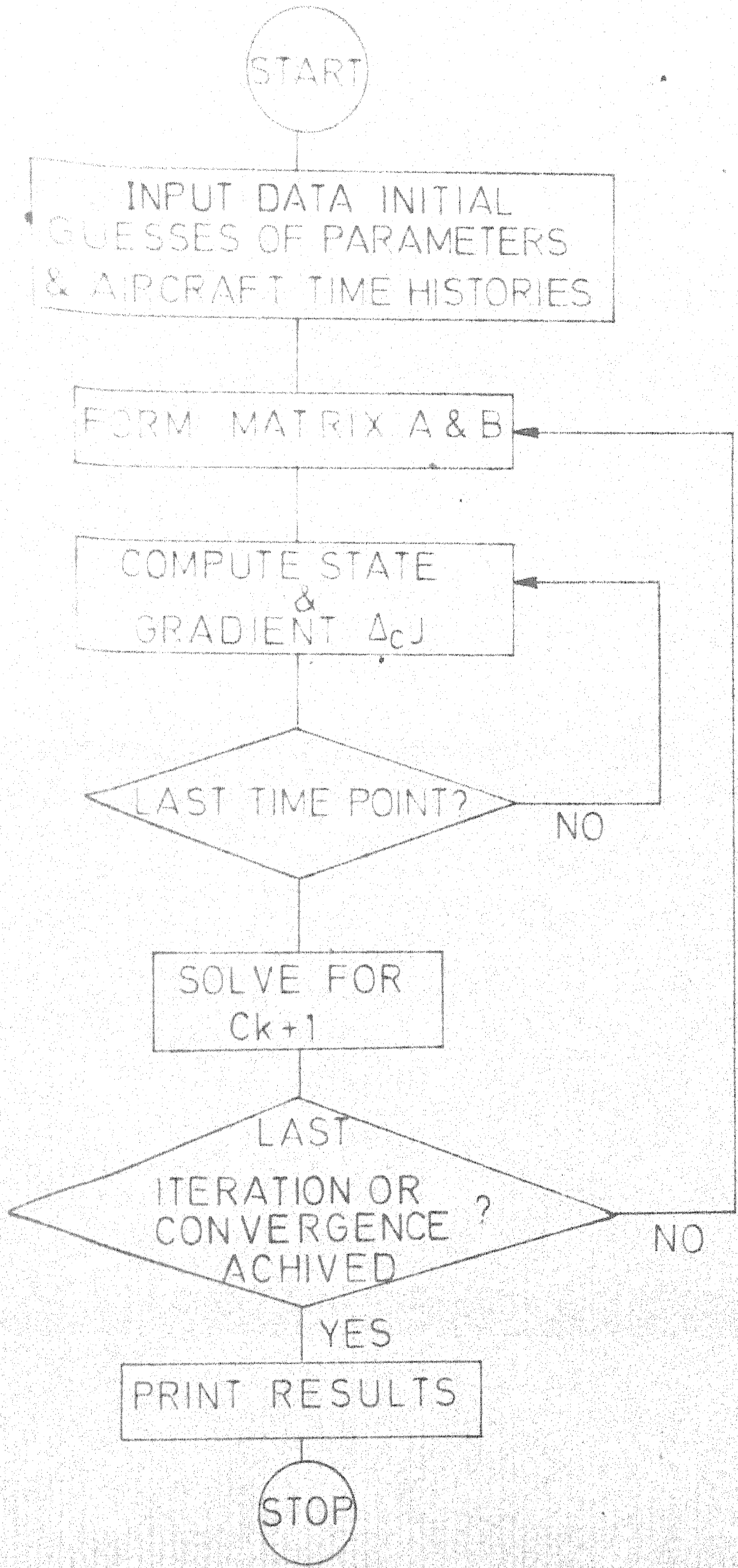


FIG-2 LOGICAL FLOW DIAGRAM FOR COMPUTER PROGRAM OF MODIFIED NEWTON RAPHSON TECHNIQUE & DAVIDON-FLETCHER-POWELL TECHNIQUE

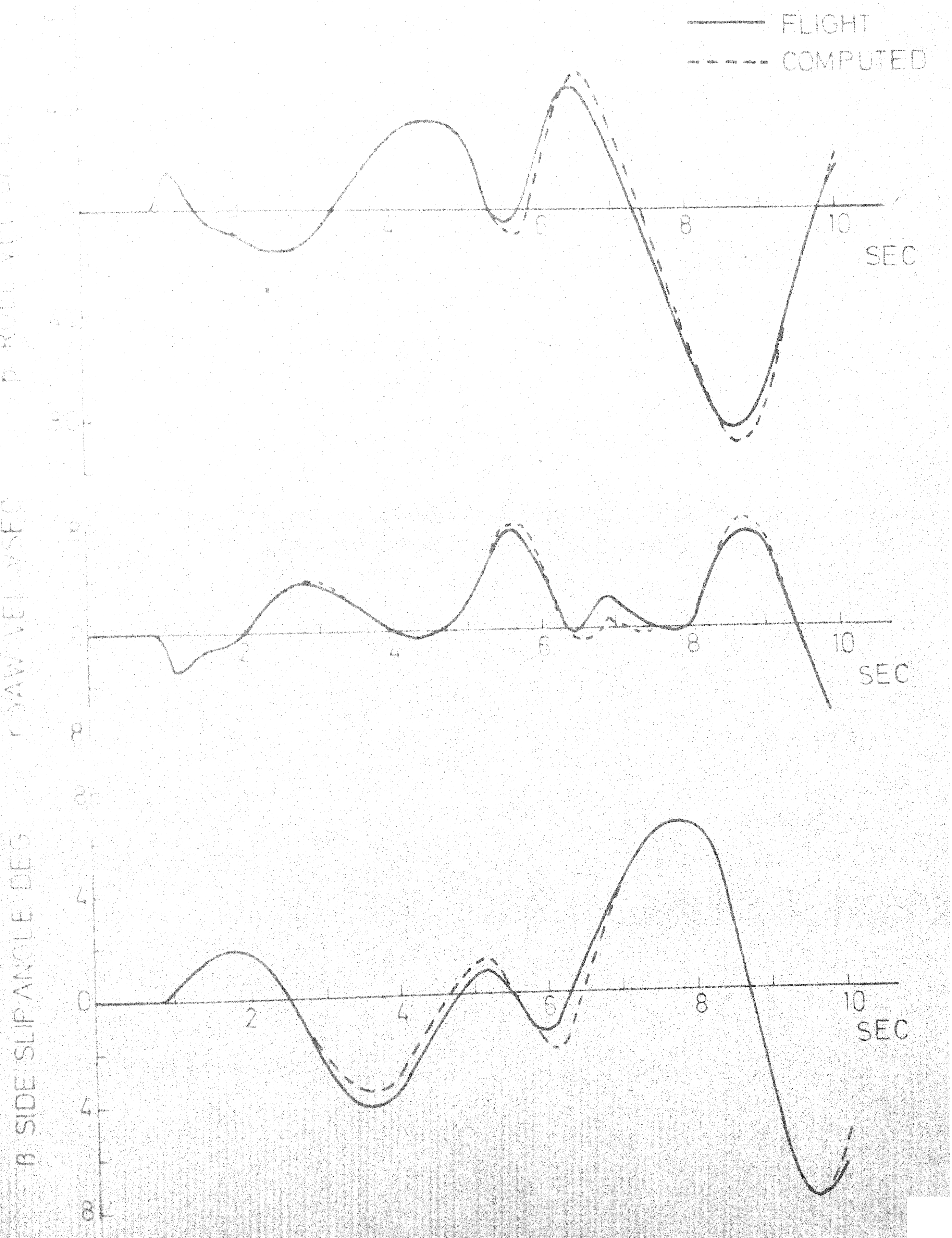
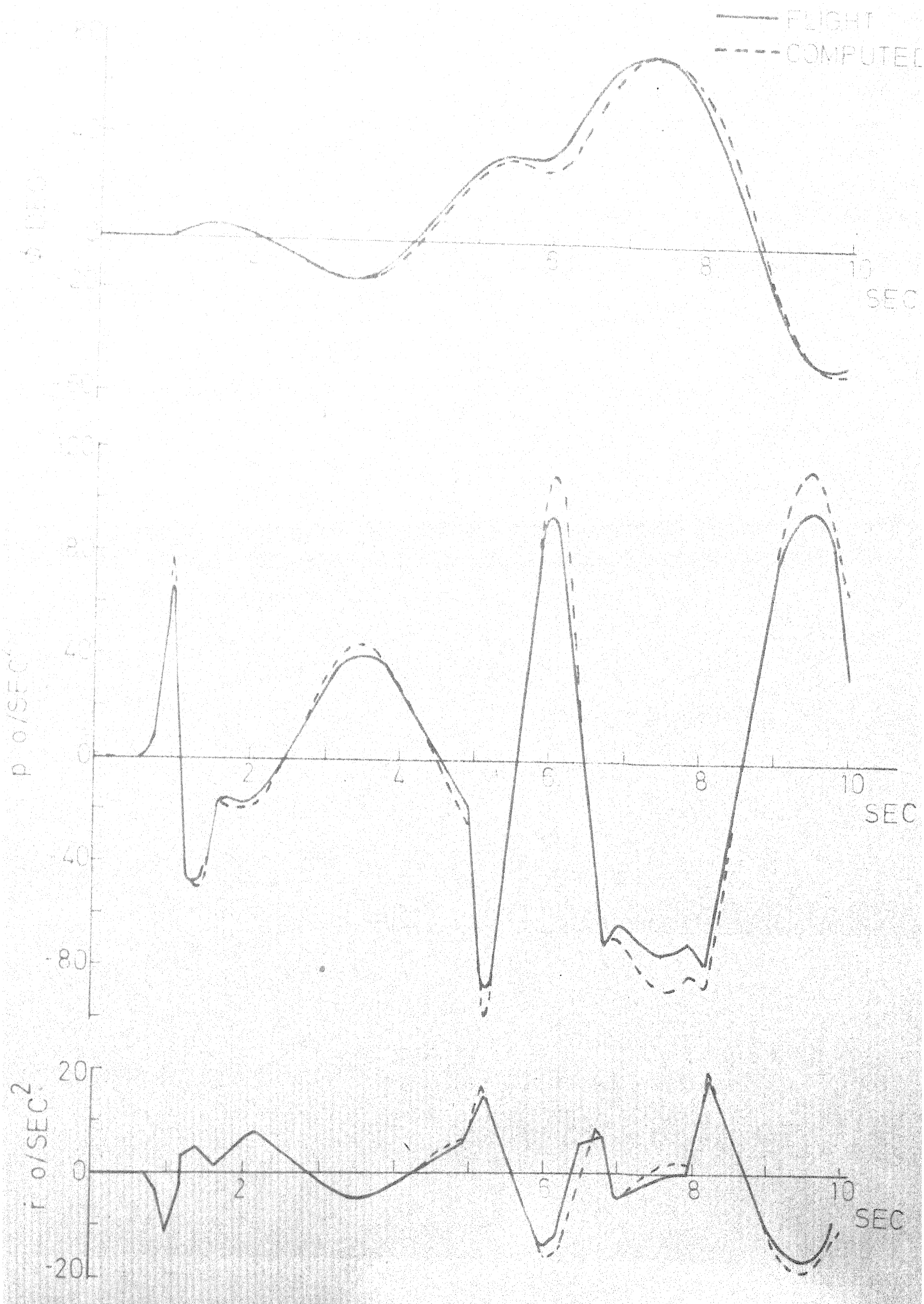


FIG.3 COMPARISON OF TIME HISTORIES MEASURED IN FLIGHT & COMPUTED BY MODIFIED NEWTON RAPHSON METHOD



FLIGHT
COMPUTED
SEC
p o/SEC
120
80
40
0
-40
-80
2
4
6
8
10
SEC
ṙ o/SEC²
20
0
-20
2
6
8
10
SEC

FIG 3 (CONT)

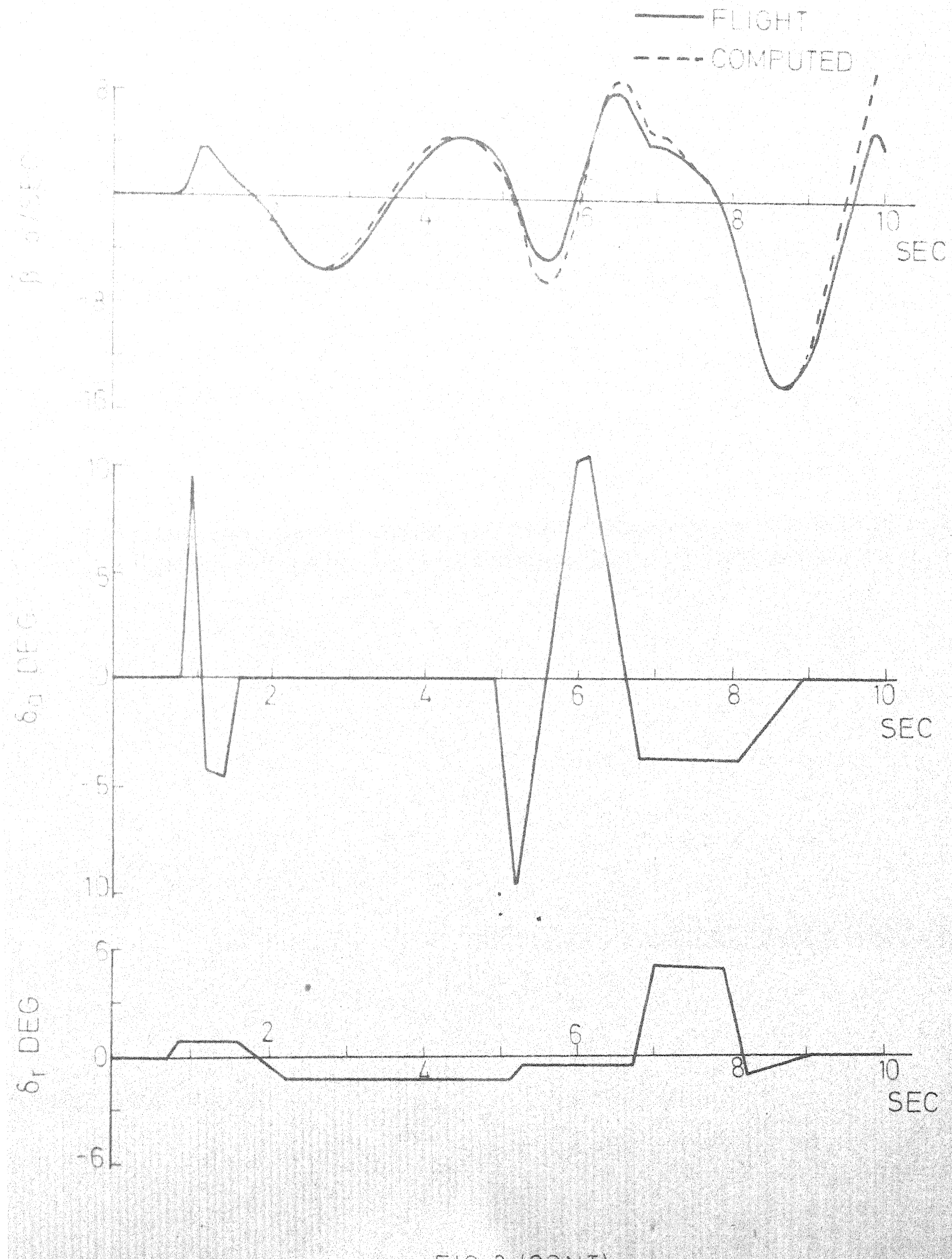


FIG 3 (CONT)

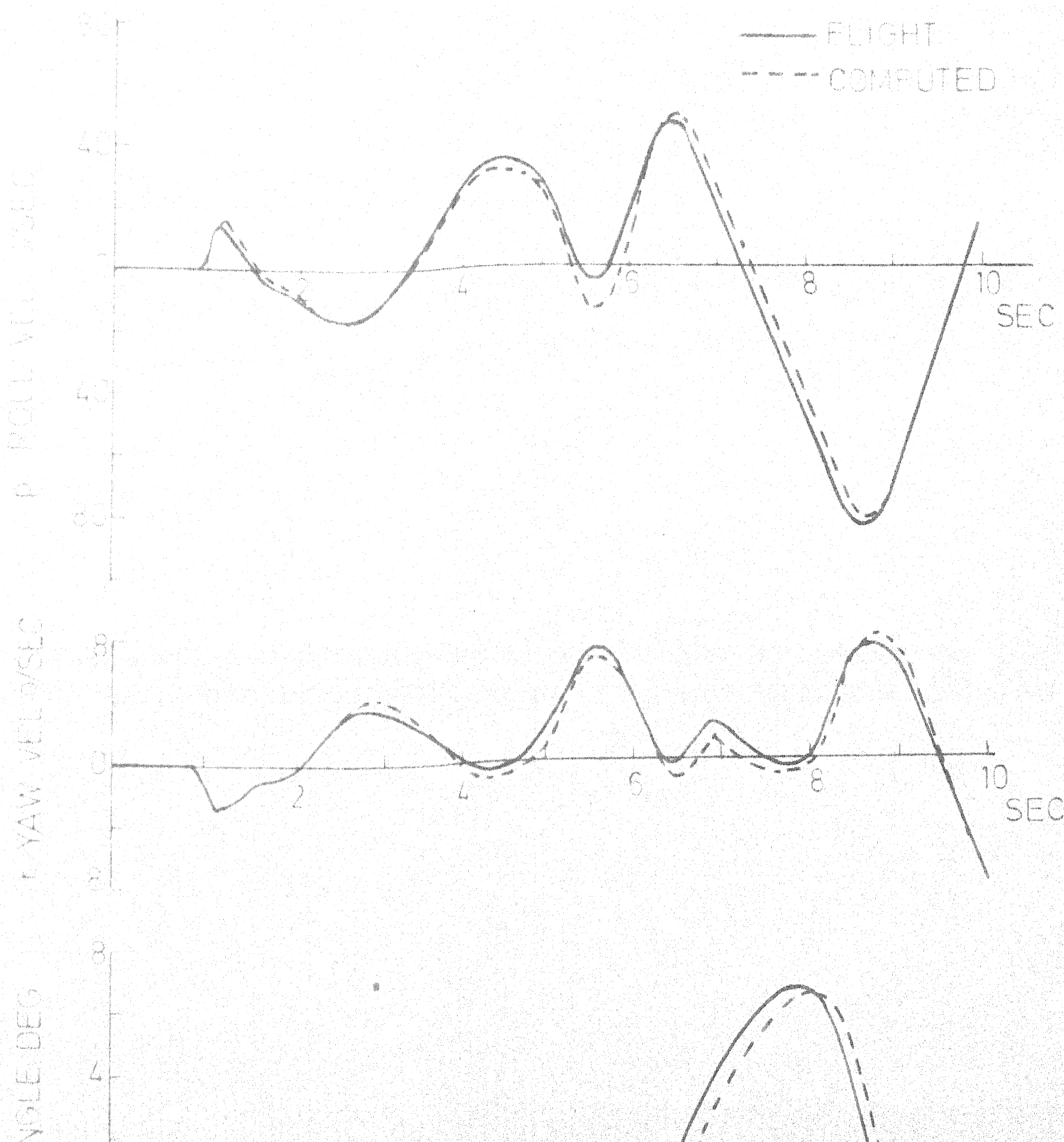


FIG 4 COMPARISON OF TIME HISTORIES MEASURED IN FLIGHT & COMPUTED BY D.F.P. METHOD

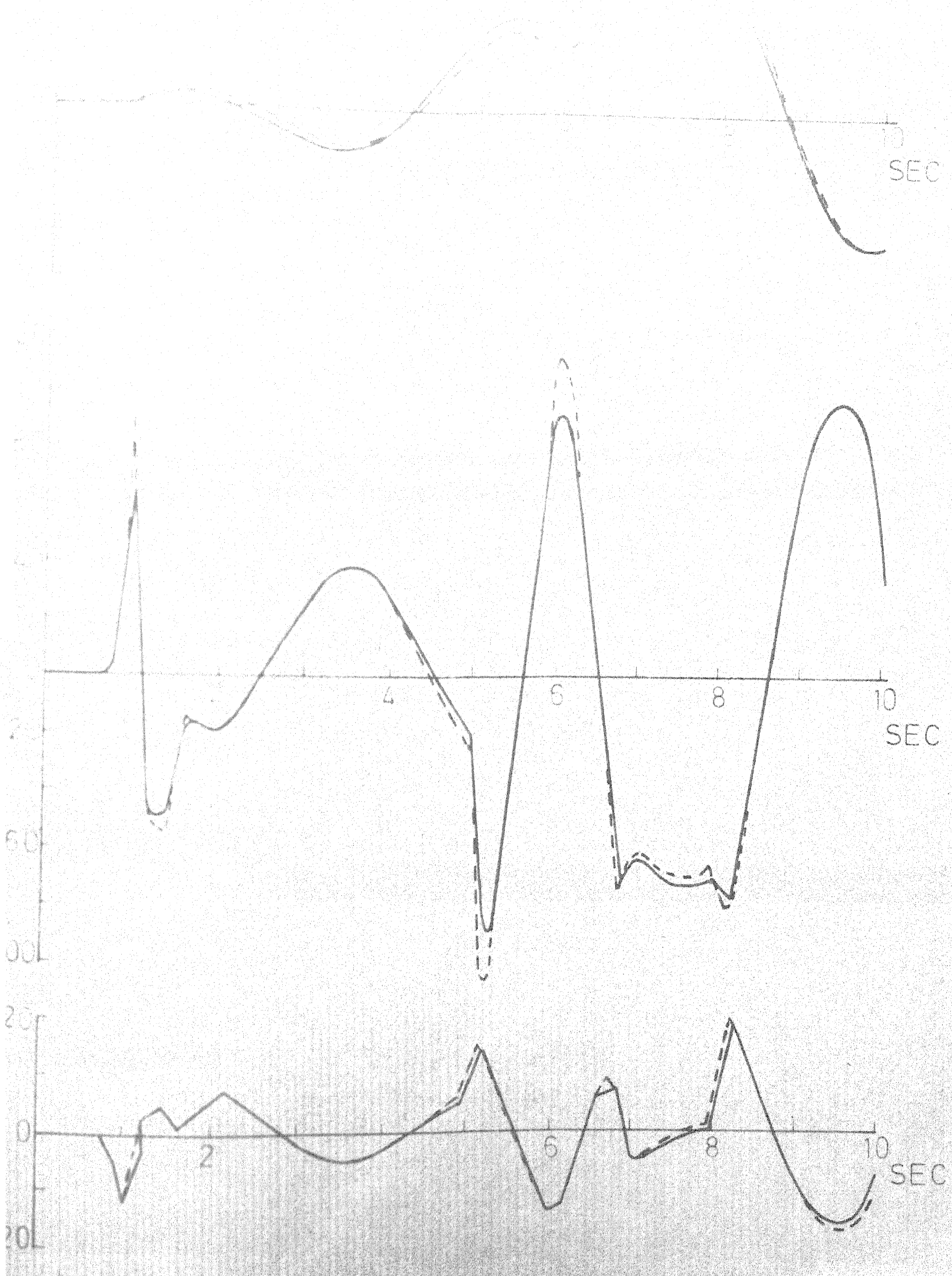


FIG.4 (CONT)

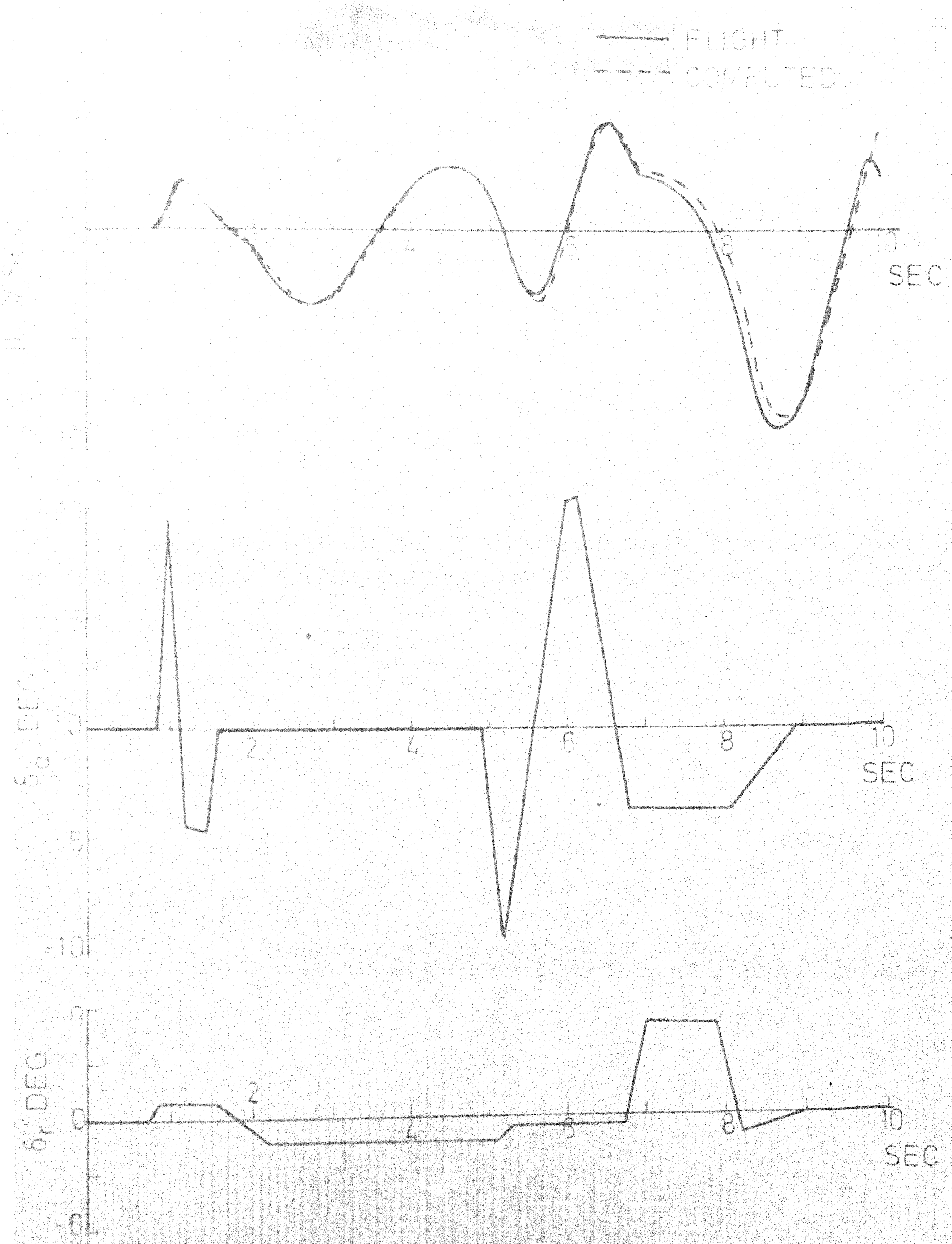


FIG.4 (CONT)